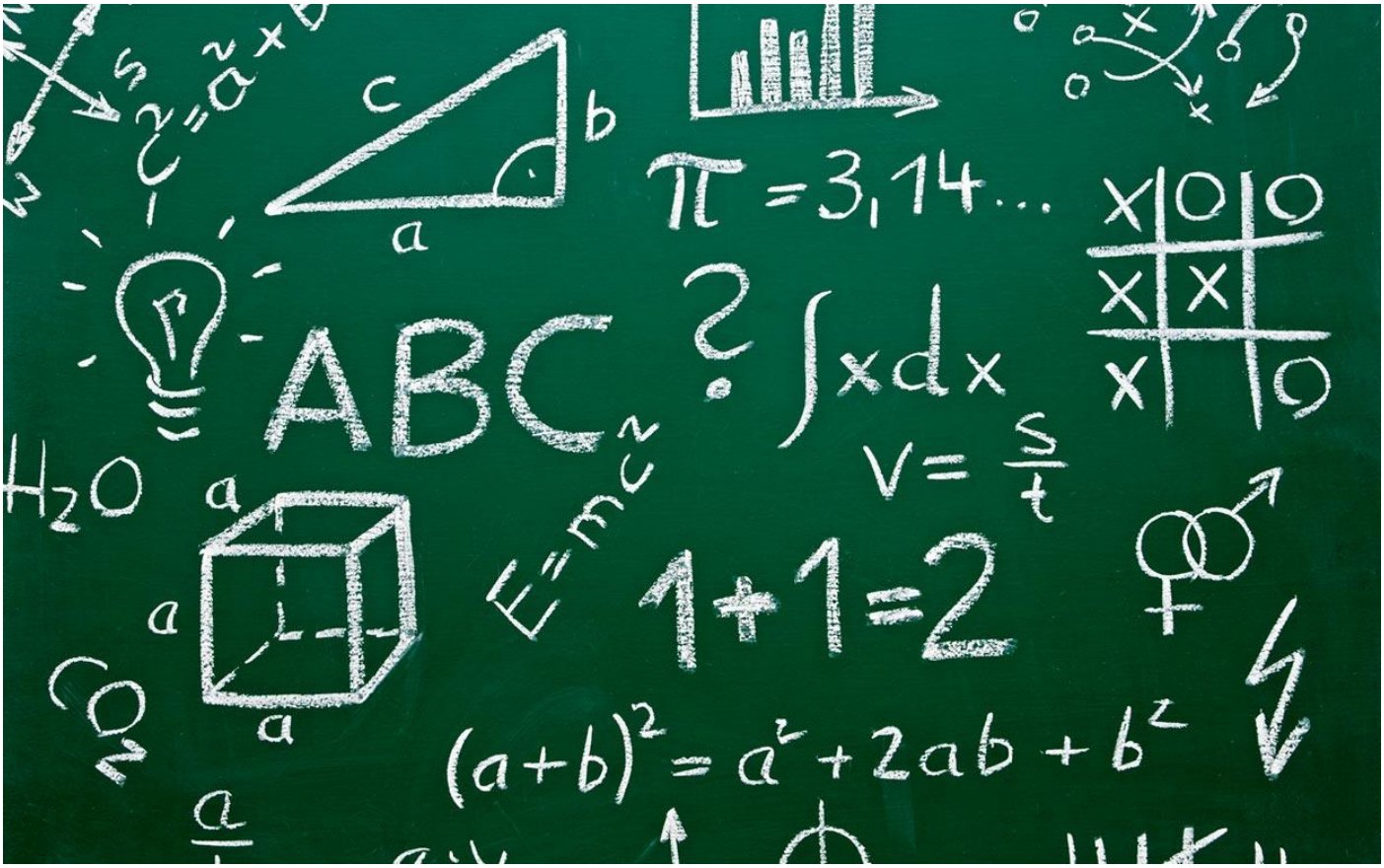
Written Calculation Policy
Barmby Moor Primary School
Garton on the Wolds Primary School

Updated January 2018



Progression towards a standard written method of calculation

Introduction

This calculation policy has been written in line with the programmes of study taken from the National Curriculum for Mathematics (2014) and Development Matters (2012) and the Early Learning Goals. It provides guidance on appropriate calculation methods and progression. It also provides guidance on how to deliver: concrete, pictoral and abstract methods to pupils and conceptual variation using NCETM White Rose Maths Hub guidance. The content is set out in yearly blocks under the following headings: addition, subtraction, multiplication and division.

Statements taken directly from the programmes of study are listed in bold at the beginning of each section.

Children will use mental methods as their first port of call when appropriate, but for calculation that they cannot do in their heads, they will need to use an efficient written method accurately and with confidence.

Aims of the Policy

- To ensure consistency and progression in our approach to calculation
- To ensure that children develop efficient, reliable, formal written methods of calculation for all operations
- To ensure that children can use these methods accurately with confidence and understanding

How to use this Policy

- Use the policy as the basis of your planning but ensure you use previous or following years' guidance to allow for personalised learning
- Always use Assessment for Learning to identify suitable next steps in calculation for groups of children
- If, at any time, children are making significant errors, return to the previous stage in calculation
- Always use suitable resources, models and images to support children's understanding of calculation and place value, as appropriate
- Encourage children to make sensible choices about the methods they use when solving problems
- Begin with concrete strategies before moving onto pictoral and then abstract strategies to secure understanding and build confidence
- Present questions in a variety of different ways to provide conceptual variation

Addition Strategies

Key vocabulary

Addition, add, total, plus, altogether, sum, addend, more than, increase, more

Addends – the numbers in a number sentence which you add together

Total – the answer to an addition question

45 + **37** = **82**

Addition strategies progression

EYFS	Y1	Y2	Y3	Y4	Y5	Y6
Counting and combining sets						
Counting on/frog (number line)	(and 100 square)			(to add with time)	(to bridge zero e.g. -11 + 14 = 3)	(to bridge zero e.g. -11 + 14 = 3)
	Bar modelling (interlocking cubes and pictures)					
		Using number facts to find related facts				
		Partitioning and recombining (dienes)				
		Expanded column addition (dienes)				
			Compact column addition			
				Rounding and adjusting		

Subtraction Strategies

Key vocabulary

Subtraction, subtract, minus, less than, take away, find the difference, deficit, reduce, minuend, subtrahend, difference, deduct, less, decrease, fewer, leave

Minuend – the number in a number sentence which you will take away from

Subtrahend – the number in a number sentence which you take away

Difference – the answer to a subtraction question

122 – **72** = **50**

Subtraction strategies progression

EYFS	Y1	Y2	Y3	Y4	Y5	Y6
Taking away from a starting (concrete objects)						
Counting back (number line)	(number line and 100 square)					
	Finding the difference/frog (counting on a number line)		(when calculating with time)	(decimals)	(bridging zero)	
	Partitioning and recombining (dienes)				(decimals)	
	Bar modelling (cubes and pictures)					
		Compensation (e.g. 57 – 19 = 38 so 57 – 20 +1)				
		Using number facts (e.g. 8 – 2 = 6 so 90 – 20 = 60)				
		Expanded column subtraction (dienes)				
			Compact column subtraction			

Multiplication Strategies

Key vocabulary

Group, times, array, altogether, sets, multiply, multiplied by, grid method, multiple, product, repeated addition, equal groups, times, partition, square, factor, square number, cube number, prime number, common factor, composite number (non-prime), power of, square root, cube root

Multiplicand – the number which is to be multiplied by the…

Multiplier – the number which you multiply by (The **multiplicand** and **multiplier** are commutative)

Product – the answer to a multiplication question

12 x 6 = 72

Multiplication strategies progression

EYFS	Y1	Y2	Y3	Y4	Y5	Y6
Arrays (concrete objects)		(jottings)				
Counting in multiples (numberline)				(multiplying fractions e.g. 4 x 3/5)		
	Grouping (concrete)	Grouping (repeated addition)				
	Bar modelling (cubes and pictures	(jottings)				
			Using number facts (2 x 3 = 6 so 20 x 3 = 60 etc)			
			Grid method			
				Ladder method		
				Short multiplication		
					Long multiplication	

Division Strategies

Key vocabulary

Share, group, groups of, lots of, divide, divided by, divided into, division, left, left over, multiple, remainder, factor, quotient, divisor

Dividend – the number which is to be divided by the…

Divisor – the number which you divide by

Quotient – the answer to a division question

80 ÷ **10** = **8**

Division strategies progression

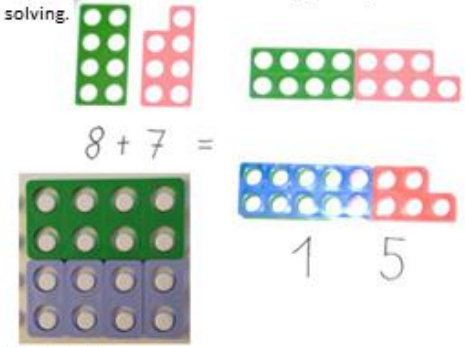
EYFS	Y1	Y2	Y3	Y4	Y5	Y6
Sharing (concrete objects)		(jottings)				
Grouping (concrete objects)		(on a numberline)	(to identify remainders)			
		Inverse relationships (2 x 5 = 10 so 10 ÷ 5 = 2 etc)		(then, 10 ÷ 5 = 2 so 10 ÷ 0.5 = 20)		
		Bar models		(finding non unit fractions of amounts)		
		Arrays				
			Bus stop method (using place value counters and then bus stop)		(bus stop, presenting remainders as decimals and fractions)	
						Long division

Equipment

Research shows that eventually, if a good model is used, then children will come to be able to work with the model, but without needing to make actual marks on paper: the model is imagined and has become a tool for thinking with.

Numicon

Numicon supports children in recognising the value or size of a number. It is also used to help children explain their mathematical thinking when problem solving.



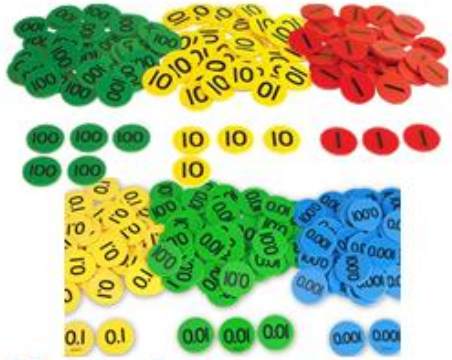
The whole is 8.
The number of equal parts is 4.
The size of each part is 2.

Dienes/ Base 10

Dienes/ base 10 are related to each other in terms of size, helping children to visualise the relative value of numbers. They are used for all four operations.

Place Value Counters

Place value counters are used to develop and consolidate understanding of place value with all four operations, and to illustrate the concept of **exchange and unitisation.** They also help children to build and decompose (spit) **numbers**



Place Value Arrows

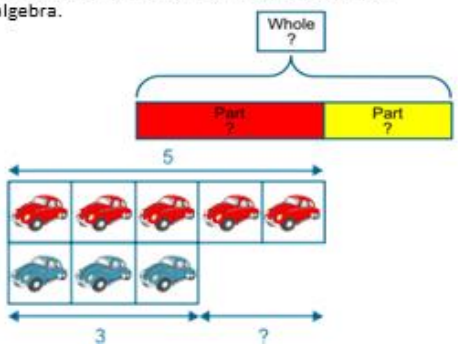
Arrow cards support place value skills and concepts and form a useful transition between base ten blocks and written numerals. Children can organise the cards horizontally or vertically to represent numbers in expanded notation. They can overlap cards and line up the arrows to form multi-digit numbers.

Bead Strings

Bead strings come in different lengths (10, 20 or 100); on a 10 bead string there are 5 red and 5 white beads whereas on a 100 bead string there are alternate groups of 10 red and 10 white beads. See appendix

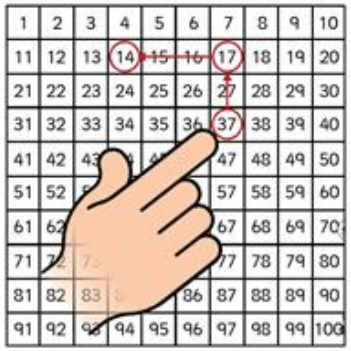
Bar Model

The Bar Model enables students to 'see' the *known* and the *unknown* (*?*) in a problem, and figure out the relationship between the two. Using this simple model drawing method, children are led from simple arithmetic to complex mathematical concepts such as decimals, fractions, ratio and algebra.



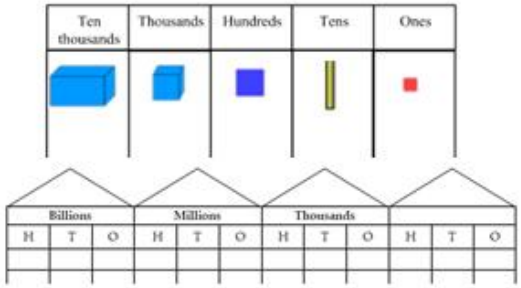
Hundred square

Children can use the grid to count up or down a certain amount of numbers and also to count ahead to find the next multiplication or sequence of numbers and to identify prime numbers.



Place Value Charts

Place Value Charts, like the ones below, are used to help children to recognise the relationship between the position of the digits in our number system.

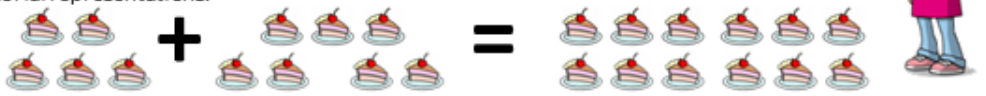


Addition **Early Years**

Add 2 single-digit numbers

Counting and Combining Sets

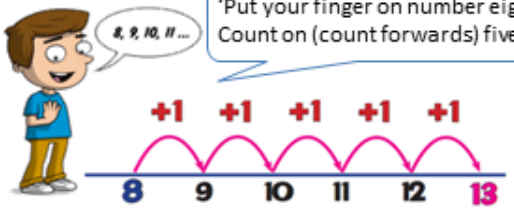
Children build on their understand of addition as counting and combining sets using concrete and pictorial representations:



Counting On

Children experience **regular counting on** from different numbers in **1s** supported first by **number tracks:** then **number lines:**

8 + 5 = 13



Children are encouraged to start with the largest number when counting on.

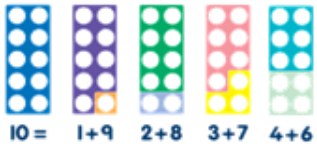
Towards a bar model

Children are encouraged to line up objects for counting and addition.

Using Number Facts

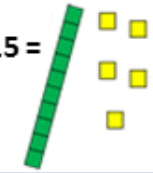
They memorise and reason with **number bonds** for numbers to **10.** They will learn that addition and subtraction are related operations:

The 'story' of 10



Partitioning

Children use bundles of straws and Dienes apparatus to model partitioning teen numbers into tens and ones and develop understanding of place value:



Playing and exploring

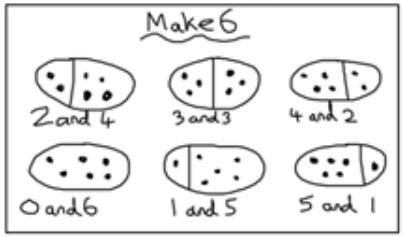
- children investigate and experience practical calculation opportunities using a wide variety of equipment, e.g. small world play, role play, counters, cubes etc.

Key skills:

- Count reliably with numbers from 1 to 20
- Place numbers in order and say which number is **one more** than a given number.
- Using quantities and objects, children will add and subtract two single-digit numbers, counting on or back to find the answer.
- They solve problems, including doubling, halving and sharing.

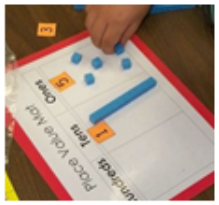
- Children practise and improve their skills in counting numbers and calculating simple addition problems.
- They are given opportunities to describe measures, using everyday language to talk about size, weight, capacity, position, distance, time and money, to compare quantities and objects and to solve problems.

Children are encouraged to develop a **mental picture** of the number system in their heads to use for calculation. They develop ways of recording calculations using pictures, etc.



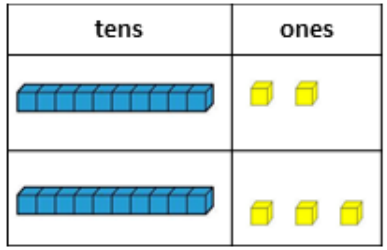
Place value

Children begin to investigate numbers using place value charts and Dienes.



They learn to understand the value of each digit and what it represents.

This enables them to effectively order and compare numbers.

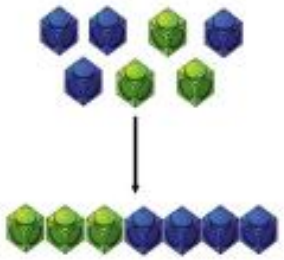
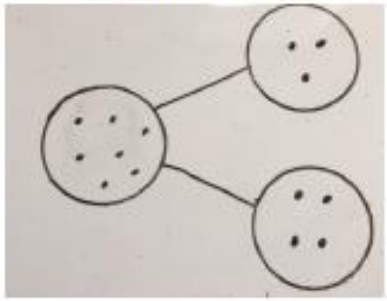
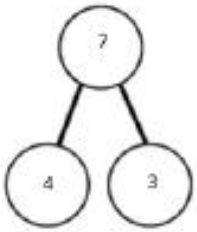
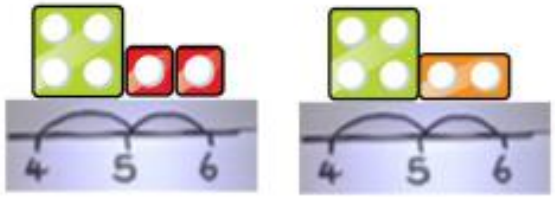
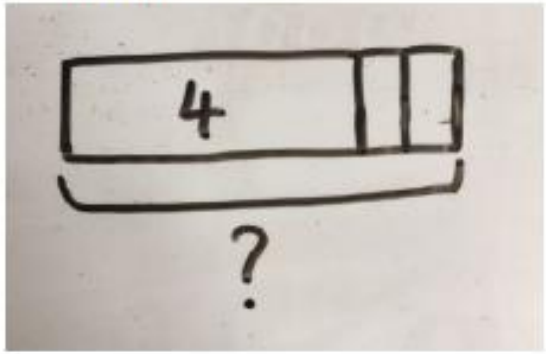


Mathematical Vocabulary

Pupils should pronounce mathematical vocabulary correctly.

addition, add, and, put together, altogether, total, equal to, equals, same as, make, more, count on, plus, sum, tens, ones, most, least, number line.

EYFS - Addition

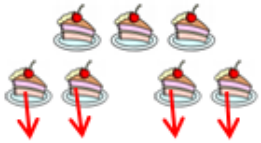
Concrete	Pictorial	Abstract
Combining two parts to make a whole (use other resources too e.g. eggs, shells, teddy bears, cars). 	Children to represent the cubes using dots or crosses. They could put each part on a part whole model too. 	4 + 3 = 7 Four is a part, 3 is a part and the whole is seven. 
Counting on using number lines using cubes or Numicon.  	A bar model which encourages the children to count on, rather than count all. 	The abstract number line: What is 2 more than 4? What is the sum of 2 and 4? What is the total of 4 and 2? 4 + 2 

Subtract 2 single-digit numbers

Subtraction as 'taking away'

Children build on their understand of taking away and counting using concrete and pictorial representations:

7 – 4 = 3

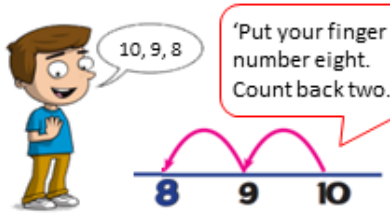


At a party, there were **7** cakes. **My friend and I ate 4. How many** cakes did we have **left?**

Counting back

Children experience **regular counting back** from different numbers in **1s** supported first by **number tracks:** then **number lines:**

10 - 2 = 8



Playing and exploring

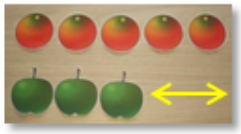
- children investigate and experience practical calculation opportunities using a wide variety of equipment, e.g. small world play, role play, counters, cubes etc.

Stories and songs

Children learn songs to develop understanding of the number system and counting back.

Towards the bar model

Children are encouraged to line up objects for counting and comparing.

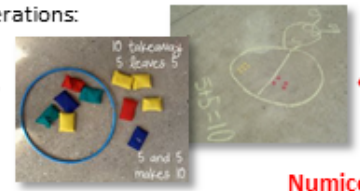


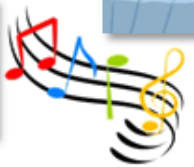
Using Number Facts

They memorise and reason with **number bonds** for numbers to **10.** They will learn that addition and subtraction are related operations:

e.g. **The 'story' of 10**

4 + 6 = 10, 10 = 6 + 4,
10 – 4 = 6, 10 – 6 = 4
4 = 10 - 6

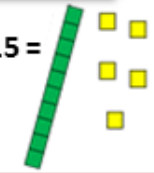




Partitioning

Children use bundles of straws and Dienes apparatus to model partitioning teen numbers into tens and ones and develop understanding of place value:

15 =



Numicon

Children use Numicon to find the difference between numbers, by covering larger tiles with smaller tiles.

Subtraction **Early Years**

Key skills:

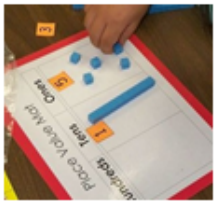
- Count reliably with numbers from 1 to 20
- Place numbers in order and say which number is **one less** than a given number.
- Using quantities and objects, children will add and subtract two single-digit numbers, counting on or back to find the answer.
- They solve problems, including doubling, halving and sharing.

- Children practise and improve their skills in counting numbers and calculating simple addition problems.
- They are given opportunities to describe measures, using everyday language to talk about size, weight, capacity, position, distance, time and money, to compare quantities and objects and to solve problems.

Children are encouraged to develop a **mental picture** of the number system in their heads to use for calculation. They develop ways of recording calculations using pictures, etc.

Place value

Children begin to investigate numbers using place value charts and Dienes.



They learn to understand the value of each digit and what it represents.

This enables them to effectively order and compare numbers.

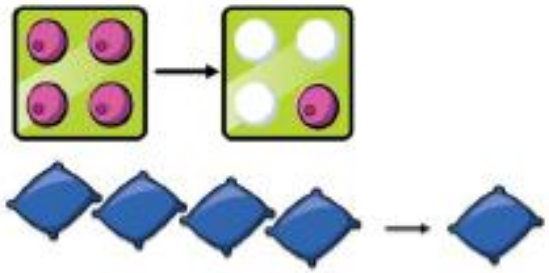
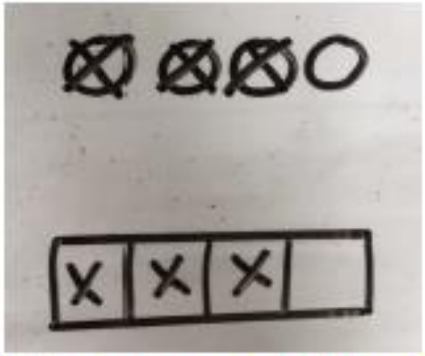
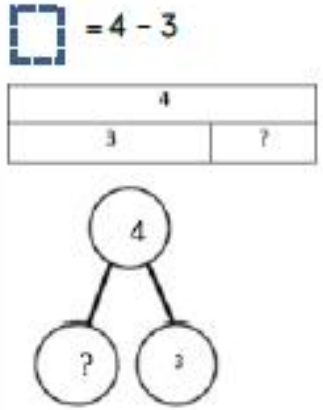
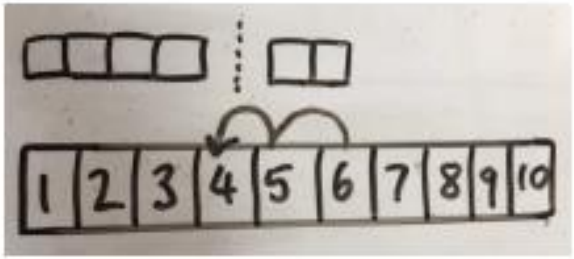
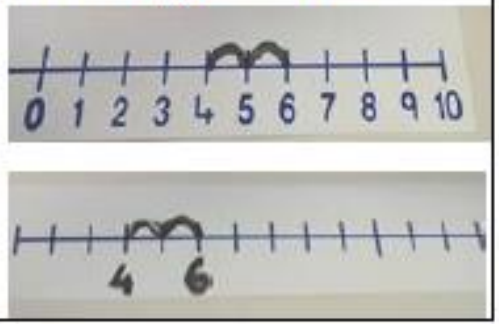
tens	ones

Mathematical Vocabulary

Pupils should pronounce mathematical vocabulary correctly.

subtract, subtraction, more than, equal to, equals, more, difference between, count back, take, take away, how many left? tens, ones, most, least, number line.

EYFS - Subtraction

Concrete	Pictorial	Abstract
Physically taking away and removing objects from a whole (ten frames, Numicon, cubes and other items such as beanbags could be used). 4 – 3 = 1 	Children to draw the concrete resources they are using and cross out the correct amount. The bar model can also be used. 	4- 3 = 
Counting back (using number lines or number tracks) children start with 6 and count back 2. 6 – 2 = 4 	Children to represent what they see pictorially e.g. 	Children to represent the calculation on a number line or number track and show their jumps. Encourage children to use an empty number line 

Multiplication as counting and doubling

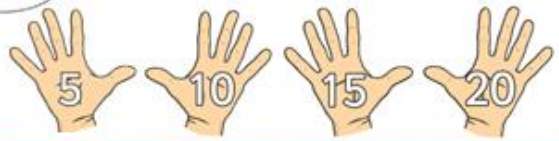
Arrays

Children investigate putting items into resources such as egg boxes, ice cube trays and baking tins which are arrays.

"2 rows of 3 or 3 rows of 2"

Counting in 2s, 5s, and 10s

Children experience **counting in amounts when grouped in 2s, 5s and 10s** this can be extended to counting on a **number tracks** then **number lines.**



Playing and exploring

Children investigate and experience practical calculation opportunities using a wide variety of equipment, e.g. small world play, role play, counters, cubes etc.

Stories and songs

Children learn songs to develop understanding of the number system and counting.

Ten fat sausages sizzling in a pan
One went 'pop' and the other went...

Mathematical Vocabulary

Pupils should pronounce mathematical vocabulary correctly.

Sets of, groups of, pair, count in twos, go together, match, adding a number to itself, double, half, twice as much, twice as long

Multiplication **Early Years**

Key skills:

- **Count reliably** with numbers from **1 to 20**
- **Place numbers in order** and say which number is **one less** than a given number.
- Using quantities and objects, children will **add** and **subtract** two **single-digit** numbers, **counting on or back** to find the answer.
- They solve problems, including **doubling**, **halving** and **sharing**.
- They are given opportunities to describe **measures**, using everyday language to talk about **size**, **weight**, **capacity**, **position**, **distance**, **time** and **money**, to **compare quantities** and **objects** and to **solve problems**

Doubling

Children begin to recognise doubles as adding a small number to itself, for example 4 + 4, to make twice as many. They will solve practical problems with doubling and learn some doubles of numbers by heart

Children are encouraged to develop a **mental picture** of the number system in their heads to use for calculation. They develop ways of recording calculations using pictures, etc.

If I add the same number, I will have double.

A child's jotting showing the fingers on each hand as a double.

Numicon

Children use Numicon to investigate doubles.

A child's jotting showing double three as three cookies on each plate.

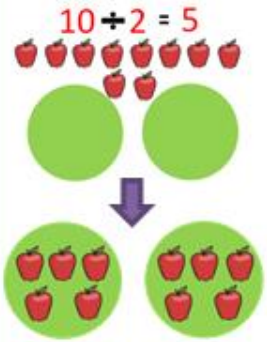
Division Early Years

Division as sharing and grouping

Children solve problems, involving doubling, halving and sharing.

Sharing

Children experience sharing physical objects out in lots of different contexts, expressing what they are doing in words and then in symbols. They are also encouraged to extend problems, e.g. "Suppose there were three people to share the apples between instead of two".



Grouping

Division as grouping is about discovering how many groups of a given number you can make out of a number of items - **"How many groups of 2 there are in 12?"** for instance.

Grouping in tens for counting.

Once confident with counting in ones and 1:1 correspondences, children will begin to group objects for counting.

Then they group in fives and count.

Mathematical Vocabulary

Pupils should pronounce mathematical vocabulary correctly.

Fair, share out, share between, share into, same number, groups, predict/guess, same number, equal, equally, equal groups,

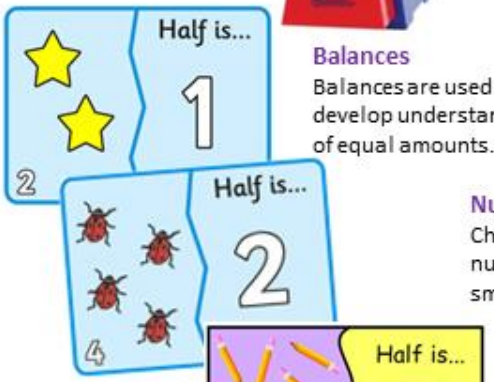
Key skills:

- **Count reliably** with numbers from **1 to 20**
- **Place numbers in order** and say which number is **one less** than a given number.
- Using quantities and objects, children will **add** and **subtract** two **single-digit** numbers, **counting on or back** to find the answer.
- They solve problems, including **doubling**, **halving** and **sharing**.
- They are given opportunities to describe **measures**, using everyday language to talk about **size**, **weight**, **capacity**, **position**, **distance**, **time** and **money**, to **compare quantities** and **objects** and to **solve problems**

Towards the concept of fractions

Doubling and halving

Children investigate doubling and halving using a wide variety of equipment, e.g. small world play, role play, counters, cubes etc.



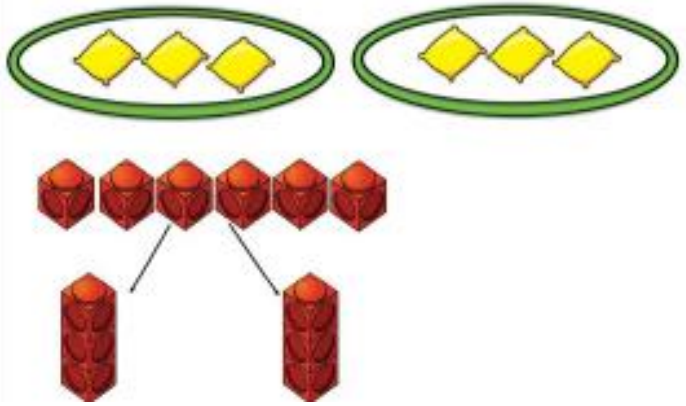
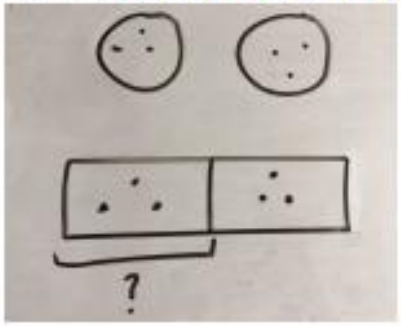
Balances

Balances are used to develop understanding of equal amounts.

Numicon

Children use Numicon to find half of a number by covering larger tiles with smaller tiles.

EYFS - Division

Concrete	Pictorial	Abstract
Sharing using a range of objects. 6 ÷ 2 	Represent the sharing pictorially. 	6 ÷ 2 = 3  Children should also be encouraged to use their 2 times tables facts.

Addition Y1

Add with numbers up to 20

Counting and Combining Sets

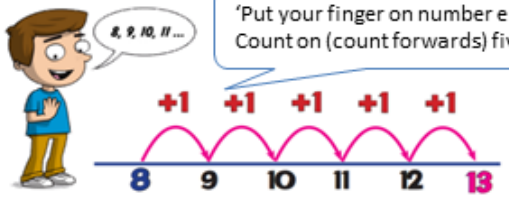
Children build on their understand of addition as counting and combining sets using concrete and pictorial representations:

12 + 5 = 17

Counting On

Children experience **regular counting on** from different numbers in **1s** and in multiples of **2**, **5** and **10**, supported first by **number tracks:** then **number lines:**

8 + 5 = 13



They will learn to **add multiples of 10** by counting on in **10**s e.g. 15 + 20 as 15, 25, 35

34	35	36
44		46
54	55	56

Children are encouraged to start with the largest number when counting on.

Bar Modelling

Children are introduced to the bar. They are encouraged to line up objects for addition using unifix cubes

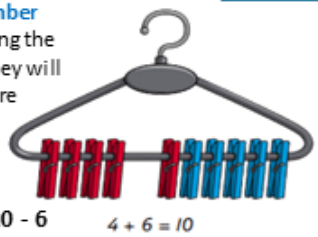
Sam had 10 red marbles and 12 blue marbles. How any marbles did he have altogether?

Using Number Facts

They memorise and reason with **number bonds** for numbers to **20,** experiencing the **equals sign** in different positions. They will learn that addition and subtraction are related operations:

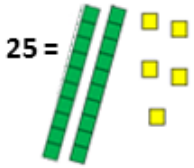
e.g. **The 'story' of 10**

4 + 6 = 10, 10 = 6 + 4,
10 – 4 = 6, 10 – 6 = 4 4 = 10 - 6



Partitioning

Children use bundles of straws and Dienes apparatus to model partitioning teen numbers into tens and ones and develop understanding of place value:



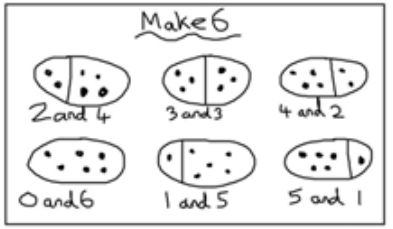
Partitioning and Recombining Numbers

Bead strings are used to illustrate addition, including **bridging through ten** by partitioning and recombining e.g. Calculating 8 + 5 by partitioning 5 into 2 and 3 then counting on 2 then counting on 3.

Key skills:

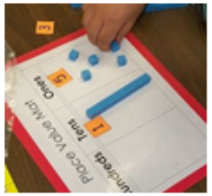
- Count to and across **100** (forwards and backwards,) from any given number
- Count, read and write numbers to **100** in numerals, including **1-20** in words
- Count in multiples of **1, 2, 5** and **10**
- Given a number, identify **one more**
- Identify and represent numbers using objects and pictorial representations including the **number line**
- Read, write and interpret mathematical statements involving addition **(+)** and equals **(=)** signs
- Represent and use number bonds and related subtraction facts within **20**
- Add **one-digit** and **two-digit** numbers to **20**, including **zero.**
- Solve simple one-step problems that involve addition and subtraction, using objects, number lines and pictorial representations,
- Solve missing number problems such as 7 = □ – 9.

Children are encouraged to develop a **mental picture** of the number system in their heads to use for calculation. They develop ways of recording calculations using pictures, etc.



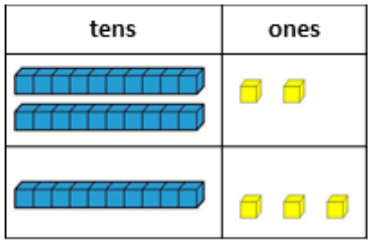
Place value

They begin to record their observations using the place value charts and Dienes.



They learn to understand the value of each digit and what it represents.

This enables them to effectively order and compare numbers.

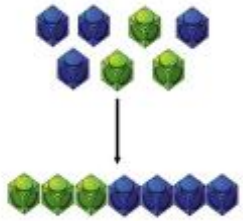
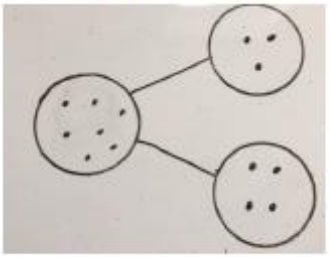
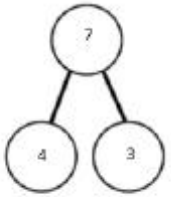
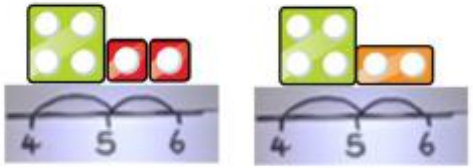
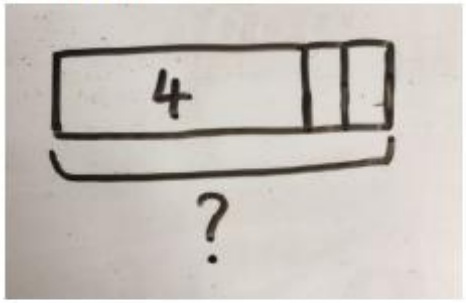
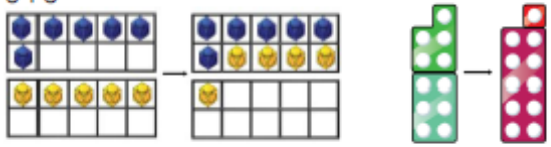
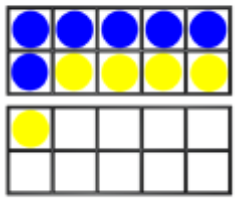


Mathematical Vocabulary

Pupils should pronounce mathematical vocabulary correctly.

addition, add, and, put together, altogether, total, equal to, equals, same as, make, more, count on, plus, sum, tens, ones, most, least, number line, partition.

YEAR ONE - Addition

Concrete	Pictorial	Abstract
Combining two parts to make a whole (use other resources too e.g. eggs, shells, teddy bears, cars). 	Children to represent the cubes using dots or crosses. They could put each part on a part whole model too. 	4 + 3 = 7 Four is a part, 3 is a part and the whole is seven. 
Counting on using number lines using cubes or Numicon.  	A bar model which encourages the children to count on, rather than count all. 	The abstract number line: What is 2 more than 4? What is the sum of 2 and 4? What is the total of 4 and 2? 4 + 2 
Regrouping to make 10; using ten frames and counters/cubes or using Numicon. 6 + 5 	Children to draw the ten frame and counters/cubes. 	Children to develop an understanding of equality e.g. 6 + □ = 11 6 + 5 = 5 + □ 6 + 5 = □ + 4

Subtract with numbers up to 20

Taking away and finding the difference

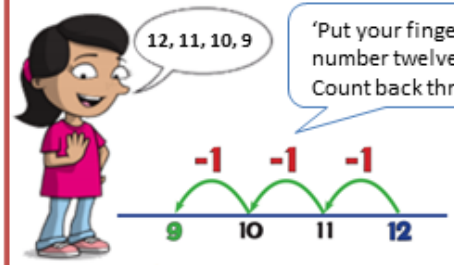
Children build on their understand of subtraction taking away to include 'finding the difference', using concrete and pictorial representations:

They learn to subtract from the total, removing objects or crossing out images on paper, and counting those remaining.

12 – 7 = 5

Counting On

Children experience **regular counting back** from a given number in **1s** and in multiples of **2, 5** and **10,** supported first by **number tracks:** then **number lines:**



12 - 3 = 9

Counting back in 10s

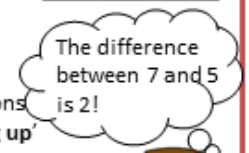
They will count back in 10s e.g. e.g. 53 – 20 as 53, 43, 33 And use this to subtract ten without counting back in 1s e.g

53 – 10 = 43

32	33	34
42	43	44
52		54

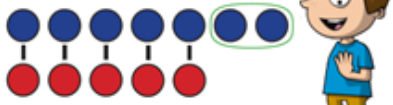
Finding the Difference

Children are introduced to the term '**difference between**' using practical apparatus, then pictorial representations.



Complementary addition

, '**counting up**' from the smallest to the largest number, is introduced **only** where the difference is small.

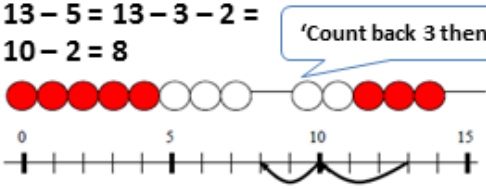


7 - 5 = 2

Partitioning and Recombining Numbers

Bead strings are used to illustrate **bridging through 10:**

13 – 5 = 13 – 3 – 2 =
10 – 2 = 8



Bar Modelling

Children are introduced to bar modelling, relating subtraction to addition through the part/ whole model.

The shop has 16 pineapples altogether. If 7 are sold, how many will be left?

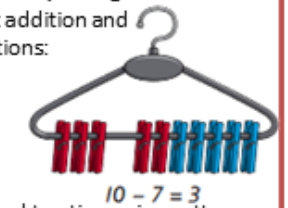
Using Number Facts

They memorise and reason with number bonds for numbers to **20,** experiencing the **equals sign** in different positions. They will learn that addition and subtraction are related operations:

e.g.

The 'story' of 10

10 – 3 = 7, 10 – 7 = 3
3 = 10 -



They will begin to investigate subtraction using patterns of known facts e.g. **7 – 3 = 4**
so we know **27 – 3 = 24, 47 – 3 = 44, 77 – 3 = 74**

Subtraction Y1

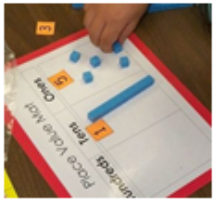
Key skills:

- Count to and across **100** (forwards and backwards,) from any given number
- Count, read and write numbers to **100** in numerals, including **1-20** in words
- Count in multiples of **1, 2, 5** and **10.**
- Given a number, identify **one less.**
- Identify and represent numbers using objects and pictorial representations including the **number line.**
- Read, write and interpret mathematical statements involving addition **(+)** and equals **(=)** signs.
- Represent and use **number bonds** and related subtraction facts within **20.**
- Subtract **one-digit** and **two-digit** numbers to **20,** including **zero.**
- Solve simple one-step problems that involve addition and subtraction, using objects, number lines and pictorial representations.
- Solve missing number problems such as 7 = □ – 9.

Children are encouraged to develop a **mental picture** of the number system in their heads to use for calculation. They develop ways of recording calculations using pictures, etc.

Understanding place value

They begin to record their observations using the place value charts and Dienes.



They learn to understand the value of each digit and what it represents.

This enables them to effectively order and compare numbers.

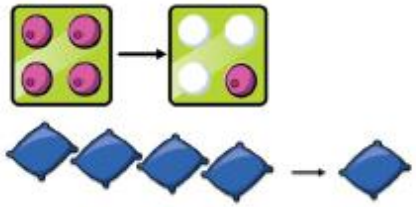
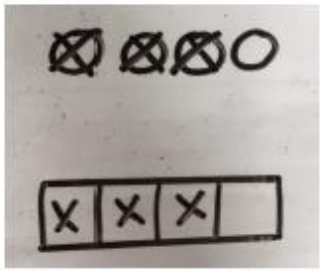
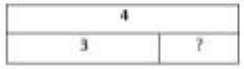
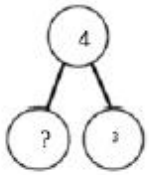
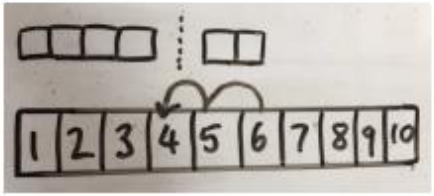
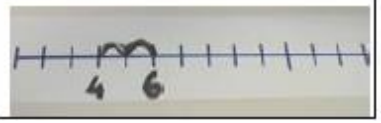
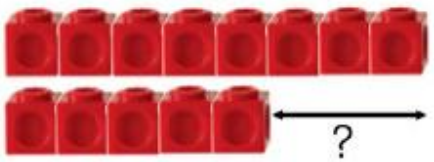
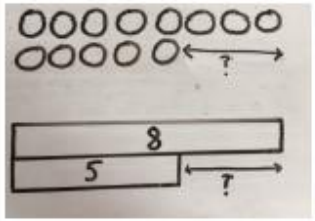
tens	ones

Mathematical Vocabulary

Pupils should pronounce mathematical vocabulary correctly.

subtract, subtraction, more than, equal to, equals, more, difference between, count back, take, take away, how many left? tens, ones, most, least, number line.

YEAR ONE - Subtraction

Concrete	Pictorial	Abstract
Physically taking away and removing objects from a whole (ten frames, Numicon, cubes and other items such as beanbags could be used). 4 – 3 = 1 	Children to draw the concrete resources they are using and cross out the correct amount. The bar model can also be used. 	4- 3 = □ = 4 – 3  
Counting back (using number lines or number tracks) children start with 6 and count back 2. 6 – 2 = 4 	Children to represent what they see pictorially e.g. 	Children to represent the calculation on a number line or number track and show their jumps. Encourage children to use an empty number line  
Finding the difference (using cubes, Numicon or Cuisenaire rods, other objects can also be used). Calculate the difference between 8 and 5. 	Children to draw the cubes/other concrete objects which they have used or use the bar model to illustrate what they need to calculate. 	Find the difference between 8 and 5. 8 – 5, the difference is □ Children to explore why 9 - 6 = 8 – 5 = 7 – 4 have the same difference.

Multiplication Y1

Multiply with concrete objects, arrays and pictorial representations

Through grouping small quantities, pupils begin to understand **multiplication** and **doubling numbers** and **quantities**. They make connections between **arrays**, **number patterns**, and **counting in twos, fives and tens**. They begin to understand that multiplication is related to combining groups of the same size (**repeated addition**).

Counting in 2s, 5s and 10s

Children are given regular experience of **counting on and back** in multiples of **2**, **5** and **10**, supported first by **number tracks** then **number lines.** They are encouraged to reason with numbers in the 2, 5 and 10 times tables.

Grouping

Children are given regular experience of **counting in equal groups** in 2s 5s and 10s **using practical objects**, such as cubes, counters, teddies, bundles of straws...

5 x 3 = 5 + 5 + 5 = 15

Arrays

Children begin to use arrays using a range of resources to develop their understanding that multiplication can be done in any order (commutative).

Through practical, problem solving activities, children experience counting equal sets or groups.
How many legs will three teddies have?

Bar Modelling

Children are introduced to bar modelling, relating multiplication to repeated addition.

5 children each have 3 apples for the picnic. How many apples are there altogether?

5 x 3 = 3 + 3 + 3 + 3 + 3 = **15**

Mathematical Vocabulary

Pupils should pronounce mathematical vocabulary correctly.

Sets of, groups of, pair, go together, adding a number to itself, double, half, twice as much, twice as long, times, array, altogether, multiply

Key skills:

- Count to and across **100** (forwards and backwards,) from any given number
- Count, read and write numbers to **100** in numerals, including **1-20** in words
- Count in multiples of **1, 2, 5** and **10.**
- solve one-step problems involving multiplication and division, by calculating the answer using **concrete objects, pictorial representations** and **arrays** with the support of the teacher.
- recognise, find and name a **half** as one of two equal parts of an object, shape or quantity
- recognise, find and name a **quarter** as one of four equal parts of an object, shape or quantity.

Doubling

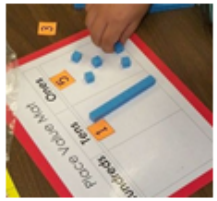
Children begin to recognise doubles as adding a small number to itself, for example 4 + 4, to make twice as many. They will solve practical problems with doubling and learn some doubles of numbers by heart. They begin to recognise multiplication as scaling in terms of doubling (e.g. 'That one tower is double the height of the other tower; it is twice the size.'.

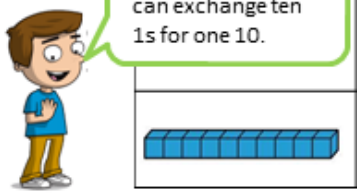
Children learn **doubles** of all numbers to 10 e.g. **double 7**

Children use their hands to work out doubles.

Place value

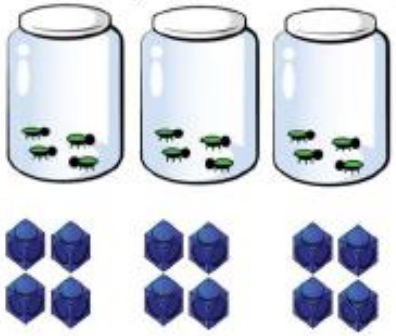
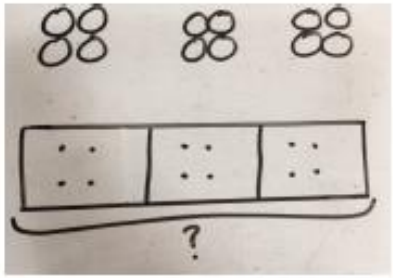
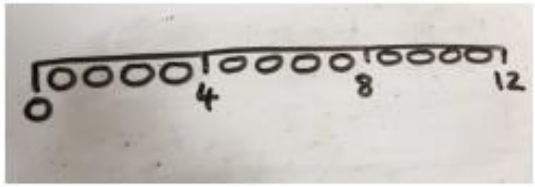
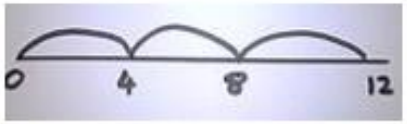
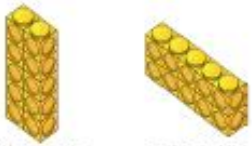
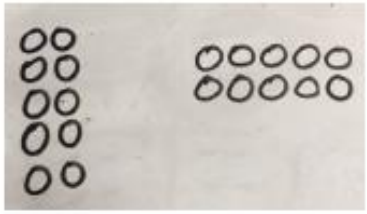
Children apply their growing knowledge of place value when doubling results in a teen number.





tens	ones

YEAR ONE - Multiplication

Concrete	Pictorial	Abstract
Repeated grouping/repeated addition 3 × 4 4 + 4 + 4 There are 3 equal groups, with 4 in each group. 	Children to represent the practical resources in a picture and use a bar model. 	3 × 4 = 12 4 + 4 + 4 = 12
Number lines to show repeated groups- 3 × 4   Cuisenaire rods can be used too.	Represent this pictorially alongside a number line e.g.: 	Abstract number line showing three jumps of four. 3 × 4 = 12 
Use arrays to illustrate commutativity counters and other objects can also be used. 2 × 5 = 5 × 2  2 lots of 5 5 lots of 2	Children to represent the arrays pictorially. 	Children to be able to use an array to write a range of calculations e.g. 10 = 2 × 5 5 × 2 = 10 2 + 2 + 2 + 2 + 2 = 10 10 = 5 + 5

Division Y1

Group and share small quantities

Through **grouping** and **sharing** small quantities, children begin to understand: multiplication and division; **doubling** numbers and quantities; and **finding simple fractions** of objects, numbers and quantities. They make connections between **arrays**, **number patterns**, and counting in twos, fives and tens. They connect halves and quarters to the equal sharing and grouping of sets of objects

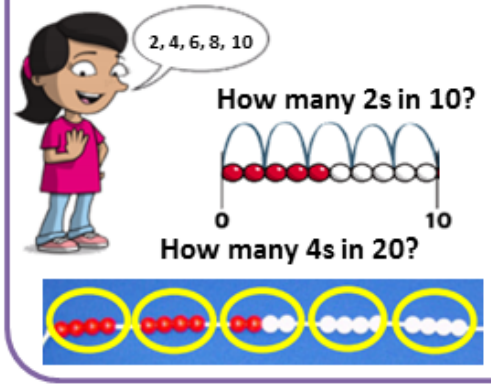
Sharing

Children develop their understanding of sharing as **equal amounts**. It is important that when making the connections to fractions that children have a range of experiences, for example, sharing liquid equally between containers; sharing food at a tea party or sharing amounts based on their mass using a balance.

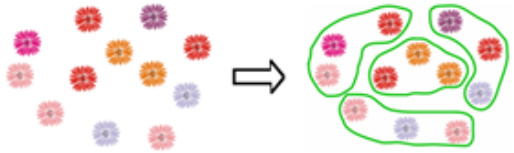
Using scales to share objects equally between two: finding half.

Grouping

Children begin to find out how many groups there are in a given amount and build on their understanding of counting in steps of 2, 5 and 10 objects.



There are 12 daisies.
How many groups of 3 can we make?



How many 3s are in 12? 4
12 divided by 3 is 4.

Mathematical Vocabulary

Pupils should pronounce mathematical vocabulary correctly.

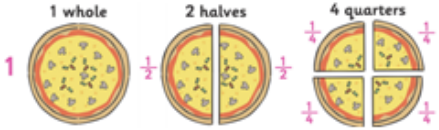
Share, share between, share equally, group, groups of, sets of, array, half, quarter

Key skills:

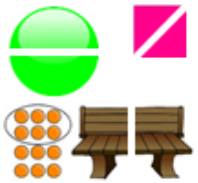
- Count to and across **100** (forwards and backwards,) from any given number
- Count, read and write numbers to **100** in numerals, including **1-20** in words
- Count in multiples of **1, 2, 5** and **10.**
- solve one-step problems involving multiplication and division, by calculating the answer using **concrete objects**, **pictorial representations** and **arrays** with the support of the teacher.
- recognise, find and name a **half** as one of two equal parts of an object, shape or quantity
- recognise, find and name a **quarter** as one of four equal parts of an object, shape or quantity.

Relating fractions to division

Children begin to recognise and combine halves and quarters as parts of a whole.



They begin to find 'fractions of' discrete and continuous quantities by solving problems using shapes, objects and quantities through a range of practical activities.

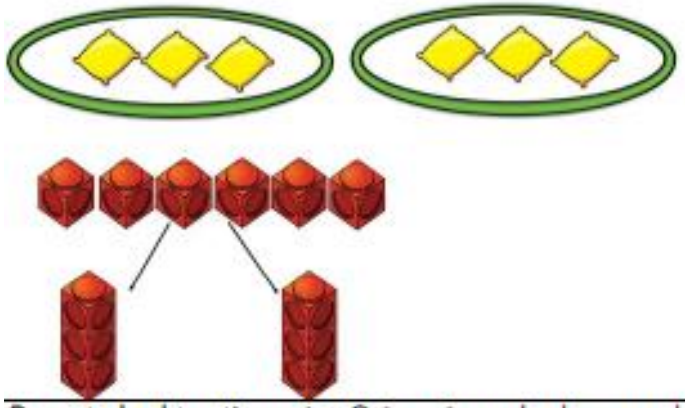
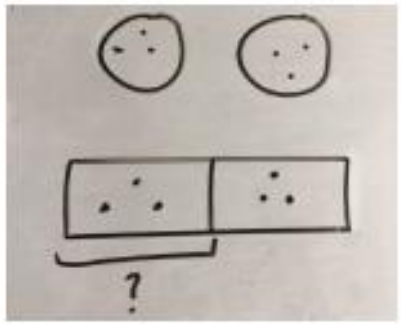
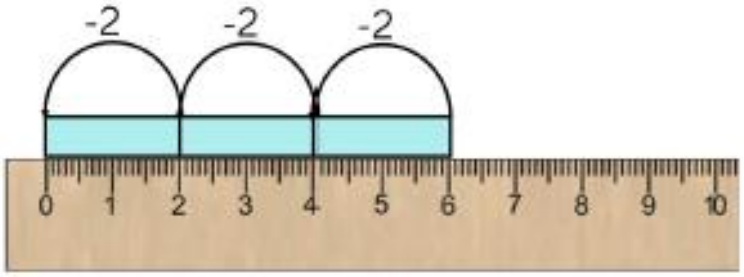
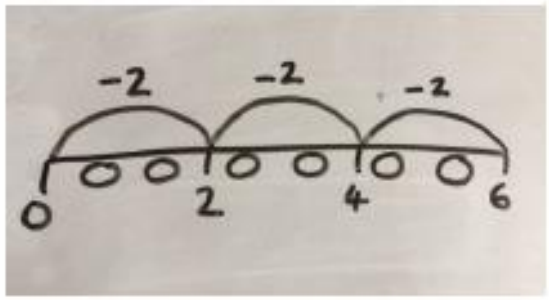
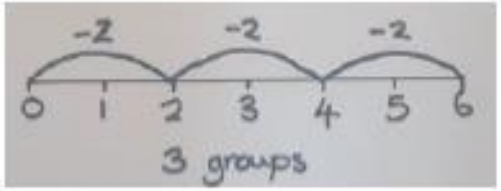
Examples	Non-examples
	
	

$\frac{1}{2}$

Pupils use the language of **time**, including telling the time throughout the day, first using o'clock and then **half** past.

Comparing and contrasting examples and non-examples of unit fractions allows children to develop their conceptual understanding.

YEAR ONE - Division

Concrete	Pictorial	Abstract
Sharing using a range of objects. 6 ÷ 2 	Represent the sharing pictorially. 	6 ÷ 2 = 3  Children should also be encouraged to use their 2 times tables facts.
Repeated subtraction using Cuisenaire rods above a ruler. 6 ÷ 2  3 groups of 2	Children to represent repeated subtraction pictorially. 	Abstract number line to represent the equal groups that have been subtracted. 

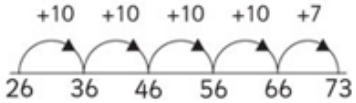
Addition Y2

Add with 2-digit numbers

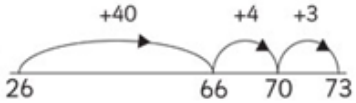
Counting on

Children will continue to **count on and back** regularly in steps of **2, 3, 5 and 10**.
They will **add 10** and **multiples of 10** to a given **1- or 2-digit number.**
They will add **two 2-digit numbers** by counting on in **10s**, then in **1s**, including **bridging through 10.**

26 + 47 = 73



OR



Using Number Facts

Children will practise addition and subtraction to 20 to become increasingly fluent in deriving facts such as, using

6 + 3 = 9, **9 – 3 = 6** and **3 = 9 – 6** to calculate **36 + 3 = 39**, **66 + 3 = 69** and **56 + 3 = 59**

Number bonds

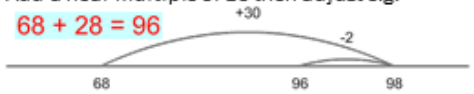
Children apply their increasing knowledge of number bonds to derive bonds to 100 (**T + T**).

8 + 2 = 10,
So, **80 + 20 = 100**

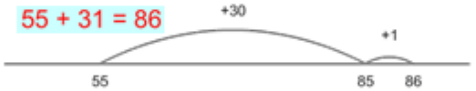
Add near multiples of 10

Add a near multiple of 10 then adjust e.g.

68 + 28 = 96

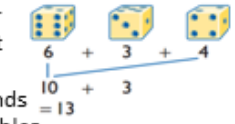


55 + 31 = 86



Doubles and Bonds

Add three or more 1-digit numbers, spotting bonds to 10 or doubles e.g.



3 + 5 + 3 = 6 + 5 = 11
6 + 3 + 4 = 10 + 3 = 13

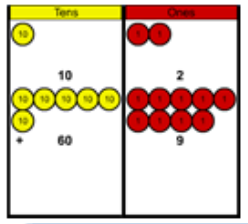
Partitioning and Recombining Numbers

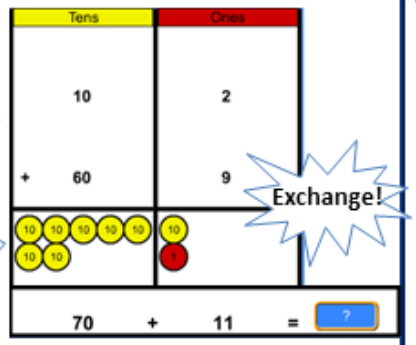
When adding with larger numbers, children use arrow cards and place value counters to support understanding of place value e.g. Partitioning **12 + 69 = ?** to get **10 + 60** and **2 + 9,** then recombining to get **70 + 11 = 81**

12 + 69 = 10 + 2
60 + 9

Partitioning

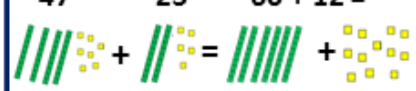
Children partition numbers in **different** ways and **recombine** using **Dienes** or **place value counters** before moving onto a formal written method e.g.





Place value counters allow children to experience the concept of '**exchanging**' ten 1s for one 10.

47 + 25 = 60 + 12 =



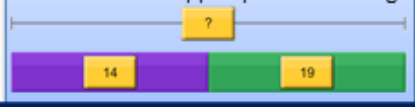
Leading to **exchange**...

= 72

Expanded method:
40 + 7
+ 20 + 5
60 + 12 = 72

Bar Modelling

The bar model illustrates the relationship between addition and subtraction to support problem solving.



Expanded addition

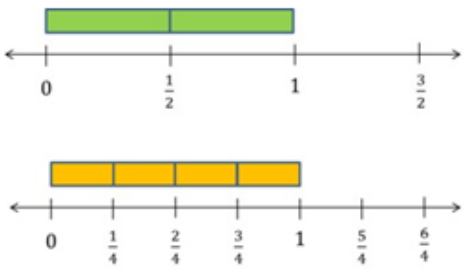
Children move to the partitioned column method once secure adding tens and units.

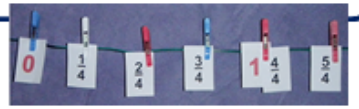
12 + 69 =

10	+	2	
60	+	9	
70	+	11	= 69

Key skills:

- Count in steps of **2, 3,** and **5** from **0**, and in **10s** from any number (forward and backward)
- Understand the place value in two-digit number (tens and ones).
- Compare and order numbers from 0 up to 100; use <, > and = signs.
- Read and write numbers to at least 100 in numerals and in words
- Recall and use addition facts to 20 fluently, and derive and use related facts up to 100
- Add numbers using concrete objects, pictorial representations, and mentally, including:
  - adding a 2-digit number and ones *(e.g. 27 + 6)* **TU + U**
  - Adding a 2-digit number and tens *(e.g. 23 + 40)* **TU + T**
  - Adding pairs of 2-digit numbers *(e.g. 35 + 47)* **TU + TU**
  - Adding three single-digit numbers *(e.g. 5 + 9 + 7)* **U + U + U**
- Show that addition can be done in any order (the **commutative** law)
- Recognise and use the **inverse** relationship between addition and subtraction to check calculations and solve **missing number problems.**
- Solve problems using objects and pictorial representations, including those involving numbers, quantities and measures applying mental and written methods.





Counting with Fractions

Children should count in fractions up to 10, starting from any number and using equivalence on the number line to reinforce the concept of fractions as numbers and that they can add up to more than one.

Mathematical Vocabulary

Pupils should read, spell and pronounce mathematical vocabulary correctly.

add, addition, more, plus, altogether, total, equal to, equals, more, and, make, sum, double, count on, column, tens, ones, most, least, number line, partition, tens boundary

YEAR TWO - Addition

Use with a 100 square to show jumping across (units) and jumping down (tens).

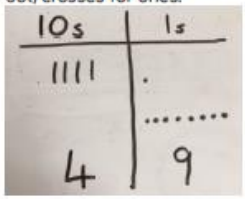
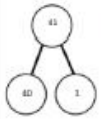
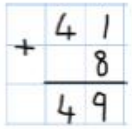
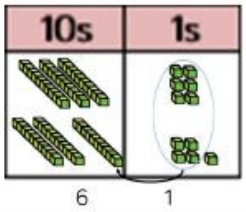
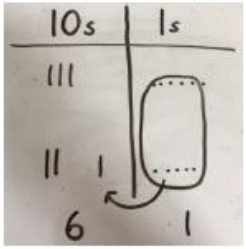
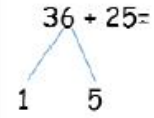
1	2	3	4	5	6	7	8	9	10
11	12	13	14	15	16	17	18	19	20
21	22	23	24	25	26	27	28	29	30
31	32	33	34	35	36	37	38	39	40
41	42	43	44	45	46	47	48	49	50
51	52	53	54	55	56	57	58	59	60
61	62	63	64	65	66	67	68	69	70
71	72	73	74	75	76	77	78	79	80
81	82	83	84	85	86	87	88	89	90
91	92	93	94	95	96	97	98	99	100

16 + 3 = 19

34 + 20 = 54

75 + 13 = 88

This would start with… 75 + 3 then 78 + 10 (partitioning)

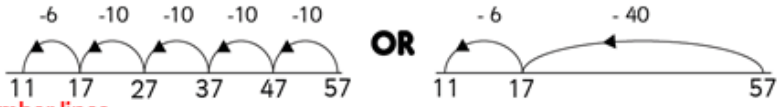
Concrete	Pictorial	Abstract
TO + O using base 10. Continue to develop understanding of partitioning and place value. 41 + 8 	Children to represent the base 10 e.g. lines for tens and dot/crosses for ones. 	41 + 8  1 + 8 = 9 40 + 9 = 49 
TO + TO using base 10. Continue to develop understanding of partitioning and place value. 36 + 25 	Chidlren to represent the base 10 in a place value chart. 	Looking for ways to make 10.  30 + 20 = 50 5 + 5 = 10 50 + 10 + 1 = 61 36 Formal method: +25 61 1

Subtract with 2-digit numbers

Counting back

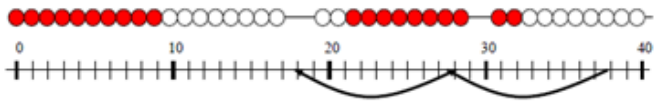
Children will continue to **count on and back** regularly in steps of **2, 3, 5 and 10**.
They will **subtract 10** and **multiples of 10** to a given **1- or 2-digit number.**
They will subtract **two 2-digit numbers** by counting back in **10s**, then in **1s**.

57 + 46 = 11



Bead strings and number lines

Beads strings are used to illustrate counting back in 10s. Number lines are used to support children's thinking e.g. **38 – 20 = 18**



Number bonds

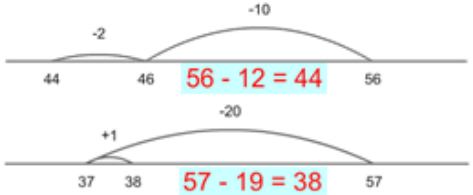
Children apply their increasing knowledge of number bonds to derive bonds to 100.

10 – 2 = 8

So, **100 – 20 = 80**

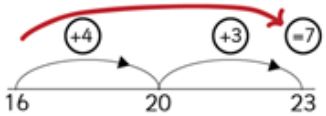
Compensation

Subtract a near multiple of 10 then adjust e.g.



Bridging through 10

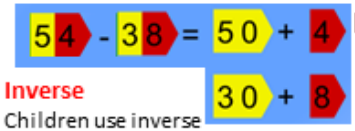
The answer can be 'built up' by counting on from the smallest to the largest number e.g. **23 – 16**

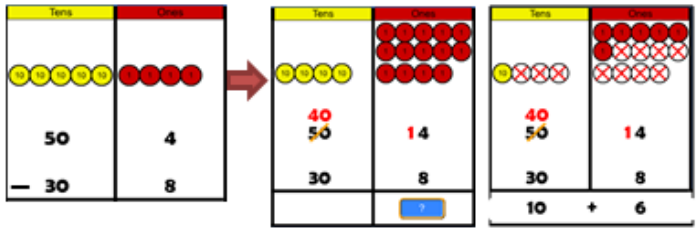


Partitioning and Recombining

Children use **arrow cards** and **place value counters** to support understanding of place value and 'exchange' e.g.

Partitioning **54 – 38** into **40 + 14** and **30 + 8** to get **40 – 30 = 10** and **14 – 8 = 6,** before recombining:

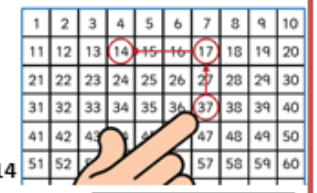




Hundred Squares

Counting back on the hundred square:

37 - 23 = ?
37 - 20 = 17
17 - 3 = 14 → 37 - 23 = 14



Inverse

Children use inverse relationships between addition and subtraction to **check** calculations and to solve **missing number problems**.

25 - 6 = ?
can be solved with...
? + 6 = 25

Expanded subtraction

Without decomposition first:

'Exchanging' one **10** for ten **1s**.

67 - 32 =

60 7
- 30 2
30 + 5 = 35

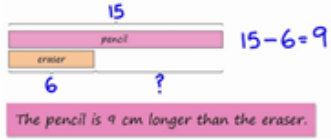
54 - 38 =

40
50 14
- 30 8
10 + 6 = 16

Exchange!

Bar Modelling

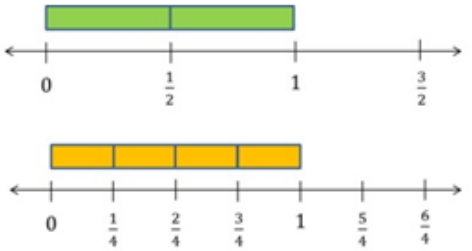
Children use bar modelling to solve one-step word problems e.g.

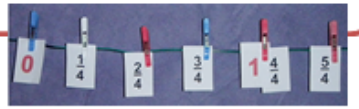


Subtraction Y2

Key skills:

- Count in steps of **2, 3,** and **5** from **0**, and in **10s** from any number (forward and backward)
- Understand the **place value** in a **2-digit** number (tens and ones)
- Compare and order numbers from **0 up to 100**; use **<**, **>** and = signs.
- Read and write numbers to at least **100** in numerals and in words.
- Recall and use addition facts to 20 fluently, and derive and use related facts **up to 100**
- Add numbers using concrete objects, pictorial representations, and mentally, including:
  - Subtracting ones from a 2-digit number *(e.g. 27 - 6)* **TU - U**
  - Subtracting tens from a 2-digit number *(e.g. 43 - 20)* **TU - T**
  - Subtracting pairs of 2-digit numbers *(e.g. 47 - 35)* **TU - TU**
- Show that subtraction of one number from another is not **commutative** (cannot be done in any order).
- Recognise and use the **inverse** relationship between addition and subtraction to **check calculations** and solve **missing number problems.**
- Solve problems using objects and pictorial representations, including those involving numbers, quantities and measures applying mental and written methods.





Counting with Fractions

Children should count in fractions up to 10, starting from any number and using equivalence on the number line to reinforce the concept of fractions as numbers and that they can add up to more than one.

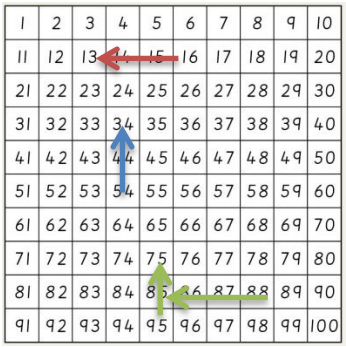
Mathematical Vocabulary

Pupils should read, spell and pronounce mathematical vocabulary correctly.

subtract, subtraction, more than, plus, altogether, total, equal to, equals, more, difference, between, count on, count back, take, take away, column, minus, how many left? tens, ones, most, least, number line, partition, tens boundary

YEAR TWO - Subtraction

Use with a 100 square to show jumping across (units) and jumping up (tens).



1	2	3	4	5	6	7	8	9	10
11	12	13	14	15	16	17	18	19	20
21	22	23	24	25	26	27	28	29	30
31	32	33	34	35	36	37	38	39	40
41	42	43	44	45	46	47	48	49	50
51	52	53	54	55	56	57	58	59	60
61	62	63	64	65	66	67	68	69	70
71	72	73	74	75	76	77	78	79	80
81	82	83	84	85	86	87	88	89	90
91	92	93	94	95	96	97	98	99	100

16 - 3 = 13

54 - 20 = 34

88 - 13 = 75

This would start with… 88 - 3 then 85 - 10 (partitioning)

Multiplication Y2

Multiply using arrays and repeated addition

(using ***at least*** 2s, 5s and 10s)

Repeated addition (grouping)

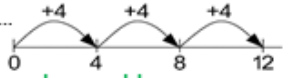
Children will continue to **count on and back** regularly in steps of **2, 3, 5 and 10**
This will include counting in 5s on a **clock face** and counting **2p, 5p and 10p coins**.
They will **solve multiplication problems** using **repeated addition** (when addition of the same number is repeated again and again) and begin to apply this when using arrays.

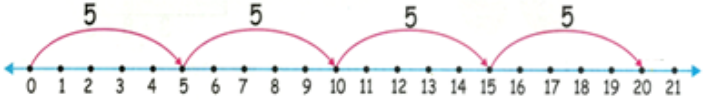
4 x 5 = 5 + 5 + 5 + 5 = 20

'Four groups of 5'

Number line

The number line continues to play an important role in developing children's understanding of multiplication as repeated addition. Starting from zero, they make equal jumps on a number line to work out multiplication facts. Empty number lines are also used...





Missing number problems

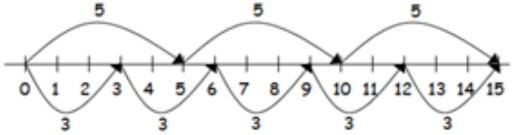
Understanding of commutativity is strengthened through the introduction of missing number problems e.g.

7 x □ = 14 **14 = 2 x □**

□ x Δ = 14 **14 = 7 x □**

□ x 2 = 14 **14 = □ x Δ**

Commutativity on a number line ...



Recording

Children express multiplication as a number sentence **using the 'x' and '=' symbols.**

Arrays

Children begin to connect the idea of counting in groups to counting the numbers in rows or columns in an array. This develops their understanding of the **commutative property** (that multiplication can be done in any order).

4 x 5 = 5 + 5 + 5 + 5 = 20

5 x 4 = 4 + 4 + 4 + 4 + 4 = 20

Mathematical Vocabulary

Pupils should read, spell and pronounce mathematical vocabulary correctly.

Groups of, sets of, times, array, altogether, multiply, count, multiplied by, repeated addition, column, row, commutative, sets of, equal groups, times, as big as, once, twice, three times

Key skills:

- Count in steps of **2, 3,** and **5** from **0**, and in **10s** from any number (forward and backward)
- Understand the **place value** in a **2-digit** number (tens and ones)
- recall and use multiplication and division **facts** for the **2, 5 and 10 multiplication tables**, including recognising odd and even numbers
- calculate mathematical statements for **multiplication** and **division** within the multiplication tables and write them using the **multiplication** (×), **division** (÷) and **equals** (=) **signs**
- show that multiplication of two numbers can be done in any order (**commutative**) and division of one number by another cannot
- solve problems involving multiplication and division, using materials, **arrays, repeated addition, mental methods**, and multiplication and division **facts**, including **problems in contexts.**

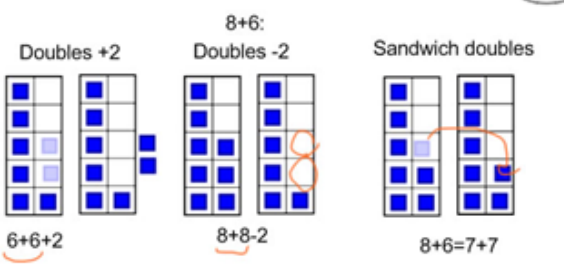
Doubles

Children investigate doubles in practical activities and begin to apply their knowledge of doubles, when adding near doubles (see diagram) and when adding three or more 1-digit numbers
e.g. identifying double 8 in...

8 + 7 + 8 = double 8 + 7 = 16 + 7 = 23

Mental Recall of times tables facts

Children should begin to **recall multiplication facts for 2, 5 and 10** times tables through practice in counting and understanding of the operation.

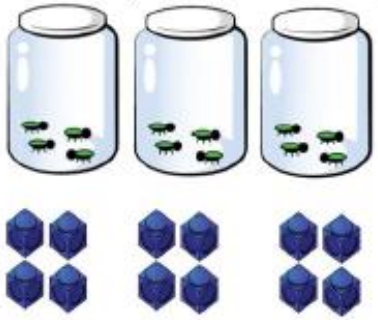
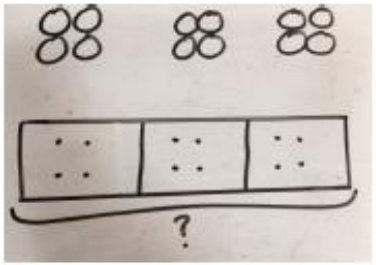
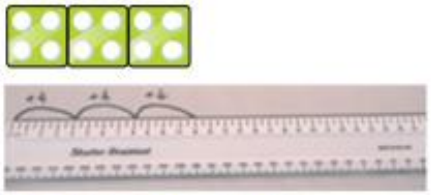
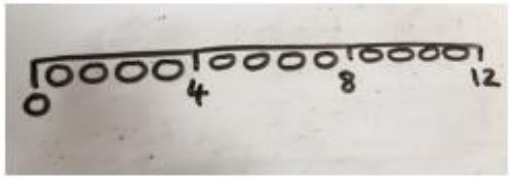
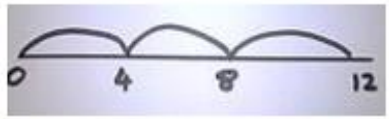
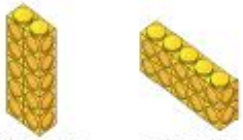
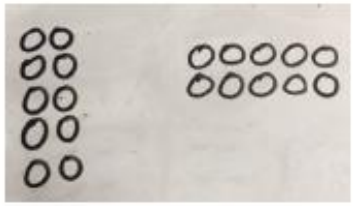


Bar Modelling

Children use bars of equal amounts to represent multiplication as repeated addition.

A Sun Racer can travel 3 cm in one second. How far can it travel in 5 seconds?

YEAR TWO - Multiplication

Concrete	Pictorial	Abstract
Repeated grouping/repeated addition 3 × 4 4 + 4 + 4 There are 3 equal groups, with 4 in each group. 	Children to represent the practical resources in a picture and use a bar model. 	3 × 4 = 12 4 + 4 + 4 = 12
Number lines to show repeated groups- 3 × 4  Cuisenaire rods can be used too.	Represent this pictorially alongside a number line e.g.: 	Abstract number line showing three jumps of four. 3 × 4 = 12 
Use arrays to illustrate commutativity counters and other objects can also be used. 2 × 5 = 5 × 2  2 lots of 5 5 lots of 2	Children to represent the arrays pictorially. 	Children to be able to use an array to write a range of calculations e.g. 10 = 2 × 5 5 × 2 = 10 2 + 2 + 2 + 2 + 2 = 10 10 = 5 + 5

Division Y2

Group and share using mathematical symbols

Children are introduced to the **multiplication tables** and use a **variety of language** to describe multiplication and division. They practise to become fluent in the **2, 5 and 10 multiplication tables** and connect them to each other. They connect the **10 multiplication table** to place value, and the **5 multiplication table** to the **divisions on the clock face**. They begin to use other multiplication tables and recall multiplication facts, including using related division facts to perform **written and mental calculations**. They work with a range of materials and contexts.

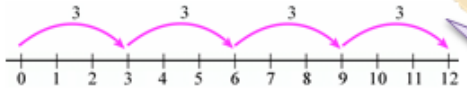
Grouping

Start with the total and make equal groups of the amount shown in the calculation. The number of groups is the answer to the calculation.

How many groups of 6 are there in 18?

Using a bead string... **18 ÷ 6**

On a number line... **12 ÷ 3**

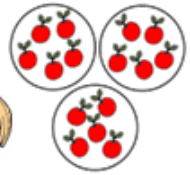


Group from zero in jumps of the divisor.

Sharing

Start with the total and share into a given number of sets. The number in each group is the answer to the calculation.

15 ÷ 3 = 5

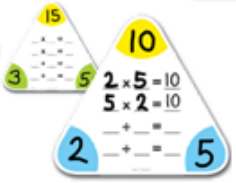


Touching the fingers in turn is a means of keeping track of how far children have counted in the sequence.

Inverse relationships

Children use commutativity and inverse relations to develop multiplicative reasoning e.g.

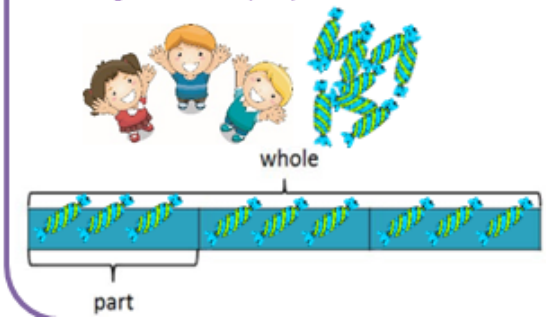
2 × 5 = 10 and **10 ÷ 5 = 2**.



Bar models

This model shows how the part and whole are related through multiplication and division. Children will use objects to share the whole amount between the parts in the model.

Dividing 9 sweets equally between 3 children



Arrays

These illustrate the inverse relationship between multiplication and division and help children to visualise a division problem. Children should understand that the objects can be grouped horizontally or vertically.

15 ÷ 3 = □
15 ÷ □ = 3
15 ÷ Δ = 5
□ ÷ 5 = 3
5 = □ ÷ 3

I can use the array to solve missing number problems.

Written division

Children will begin to use the multiplication (×), division (÷) and equals (=) signs to write mathematical statements

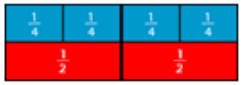
9 ÷ 3 = 3

Key skills:

- Count in steps of **2, 3,** and **5** from **0**, and in **10s** from any number (forward and backward)
- Understand the **place value** in a **2-digit** number (tens and ones)
- recall and use multiplication and division **facts** for the **2, 5 and 10 multiplication tables**, including recognising odd and even numbers
- calculate mathematical statements for multiplication and division within the multiplication tables and write them using the **multiplication** (×), **division** (÷) and **equals** (=) **signs**
- show that multiplication of two numbers can be done in any order (commutative) and division of one number by another cannot
- solve problems involving multiplication and division, using materials, **arrays, repeated addition, mental methods,** and multiplication and division **facts**, including **problems in contexts.**

Relating fractions to division

Children are given opportunities to find half, quarter and a third of shapes, objects, numbers and quantities, relating this to division.



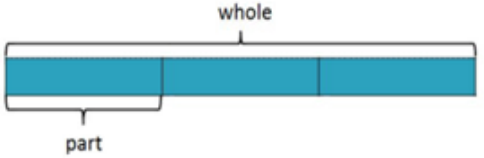
£60 boots now 1/2 price
How much for 1 pair of boots?

Finding a quarter by halving or dividing by 2 twice.

'60 ÷ 2 = 30, so 30 is a half of 60'

Bar Model

The bar diagram is used to model fraction relationships and support children in finding the correct method when solving word problems..



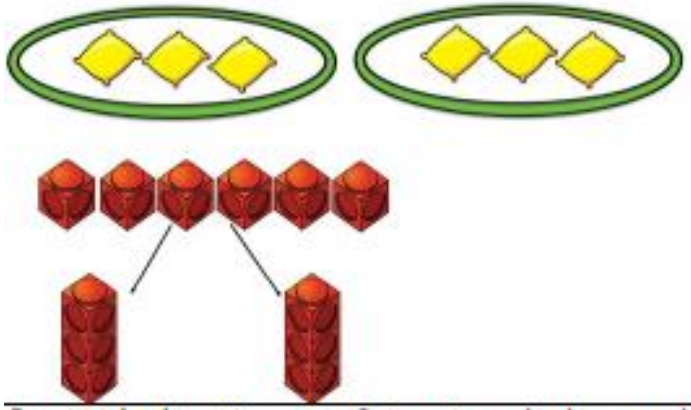
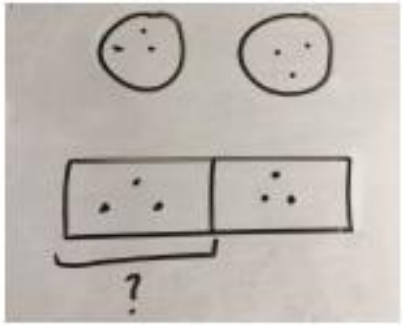
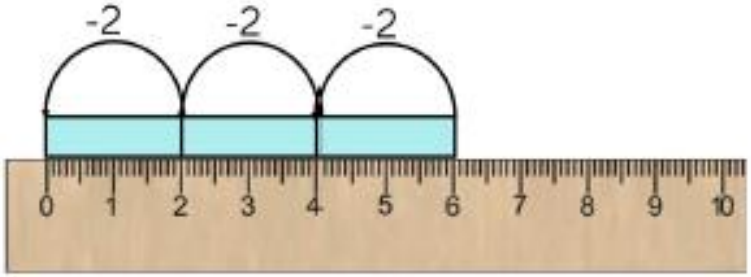
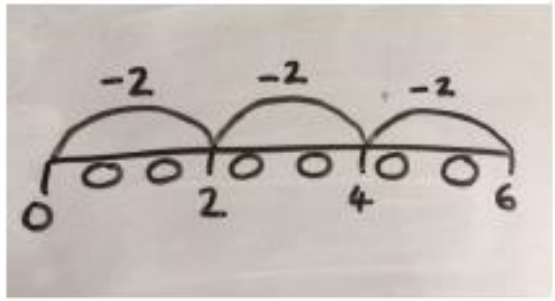
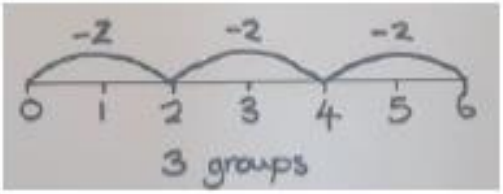
This picture shows that the 'part' is 1/3 of the 'whole'.

Mathematical Vocabulary

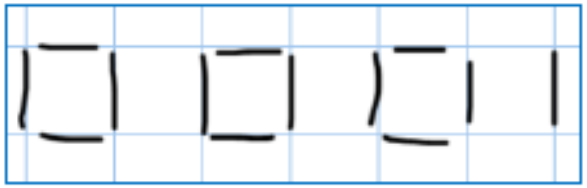
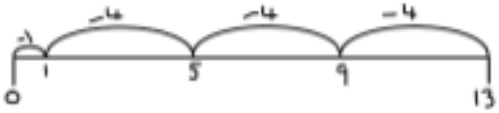
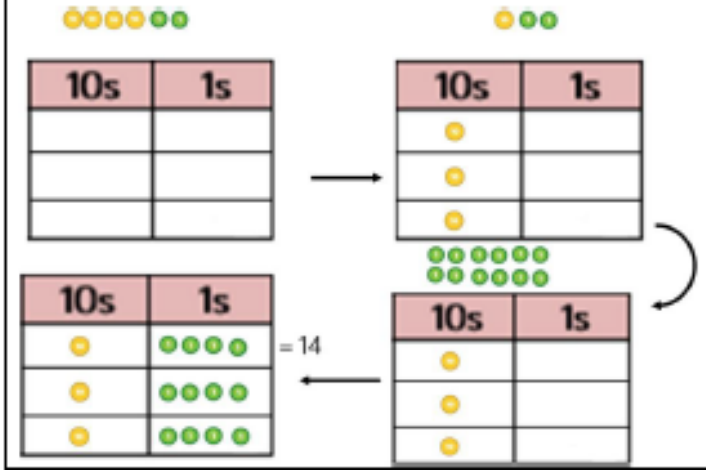
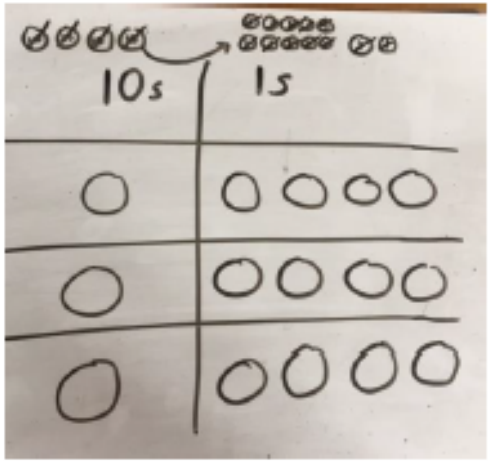
Pupils should read, spell and pronounce mathematical vocabulary correctly.

Share, share between, share equally, group, groups of, lots of, array, divide, divided by, divided into, division, left, left over

YEAR TWO - Division

Concrete	Pictorial	Abstract
Sharing using a range of objects. 6 ÷ 2 	Represent the sharing pictorially. 	6 ÷ 2 = 3  Children should also be encouraged to use their 2 times tables facts.
Repeated subtraction using Cuisenaire rods above a ruler. 6 ÷ 2  3 groups of 2	Children to represent repeated subtraction pictorially. 	Abstract number line to represent the equal groups that have been subtracted. 

YEAR TWO - Division

2d ÷ 1d with remainders using lollipop sticks. Cuisenaire rods, above a ruler can also be used. 13 ÷ 4 Use of lollipop sticks to form wholes- squares are made because we are dividing by 4.  There are 3 whole squares, with 1 left over.	Children to represent the lollipop sticks pictorially.  There are 3 whole squares, with 1 left over.	13 ÷ 4 = 3 remainder 1 Children should be encouraged to use their times table facts; they could also represent repeated addition on a number line. '3 groups of 4, with 1 left over' 
Sharing using place value counters. 42 ÷ 3 = 14 	Children to represent the place value counters pictorially. 	Children to be able to make sense of the place value counters and write calculations to show the process. 42 ÷ 3 42 = 30 + 12 30 ÷ 3 = 10 12 ÷ 3 = 4 10 + 4 = 14

Addition Y3

Add with numbers up to 3-digits

Children will continue to practise a **wide range of mental addition strategies** and use **empty number lines,** including **bridging through 100,** with increasingly large numbers. They use **compensation,** where appropriate, and begin to use more **formal written methods**.

Near Doubles

Children will use near doubles to calculate e.g.

28 + 25 =
25 + 3 + 25 =
50 + 3 = 53

167 + 153 =
150 + 17 + 150 + 3 =
300 + 17 + 3 =
300 + 20 = 320

Partitioning

Building on work in year 2, children will **partition numbers in a variety of ways**, related to place value, using varied and increasingly complex problems.

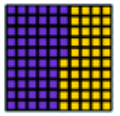
Counting on, partitioning the second number only:

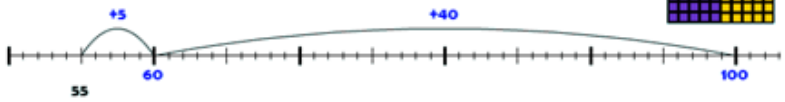
346 + 132
= 446 + 30 + 2
= 476 + 2
= 478

Number bonds

Children will develop their use of number bonds to include the 'story' of 100 (**TU + TU**):

55 + 45 = 100





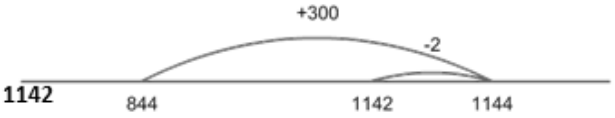
Near multiples of 10/100

The **empty number line** continues to be used where appropriate, to support mental addition of numbers that are close to a multiple of 10/100 (ending in 1, 2, 8 or 9) e.g.

844 + 298

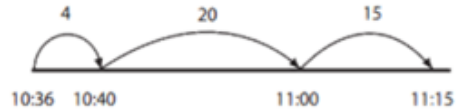
Becomes...

844 + 300 – 2 = 1142



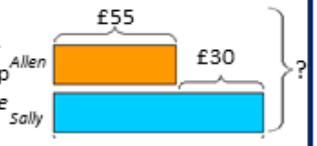
Bridging through 60 (time)

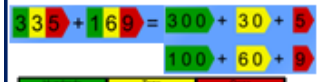
Children bridge 60 to calculate a new time e.g. Counting on 39 minutes from 10:36, bridging 10 then 60, to calculate the new time.

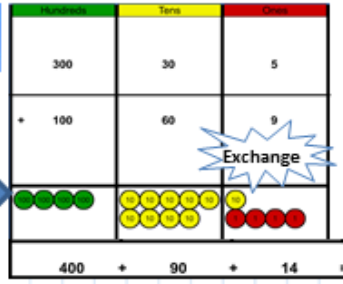


Bar modelling

Children use bar modelling to help them solve one-step word problems and begin to use it for two-step problems e.g. *Allen saved £55. Sally saved £30 more than Allen. How much did they save altogether?*







Can I make a trade?

Children use place value counters and charts to model addition, 'exchanging' ten 1s for one 10 and ten 10s for one 100.

Formal Written Method

In order to carry out this method, children need to be able to recognise the value of the hundreds, tens and ones, without recording the partitioning. Digits in the right hand column are added first. Children are reminded when using this method to say, for example **'three tens add six tens'**, not three add six.

```
335 + 169 =

300 + 30 + 5
100 + 60 + 9
400 + 90 + 14 = 504
```

```
 335
 169
  14
  90
 400
 504
   1
```

'Carry' digits underneath.

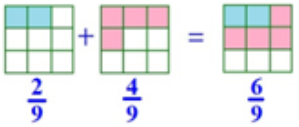
```
 335
 169
 504
  11
```

Key skills:

- Count from **0** in multiples of **4, 8, 50** and **100** and steps of **1/10.** (children will now be using multiples of **2, 3, 4, 5, 8, 50** and **100)**
- Find **10** or **100 more** than a given number.
- Recognise the **place value** of each digit in a **3-digit number** (hundreds, tens and ones)
- Compare and order numbers **up to 1000**
- Read and write numbers **up to 1000** in numerals and in words.
- Add numbers **mentally**, including:
  - a 3-digit number and ones (e.g. 127 + 6) **HTU + U**
  - a 3-digit number and tens (e.g. 223 + 40) **HTU + T**
  - a 3-digit number and hundreds (e.g. 223 + 200) **HTU + H**
- Add numbers with **up to 3-digits** using **formal written methods** of column addition.
- Estimate the answer to a calculation and use **inverse** operations to check answers.
- Solve number and practical problems, including **missing number problems**, using number facts, place value, and more complex addition.

Fractions

- Children should **count up and down in tenths;**
- Recognise and use **fractions as numbers**: unit fractions and non-unit fractions with small denominators;
- Compare and order unit fractions, and fractions with the same denominators.



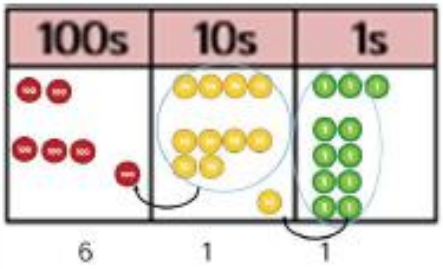
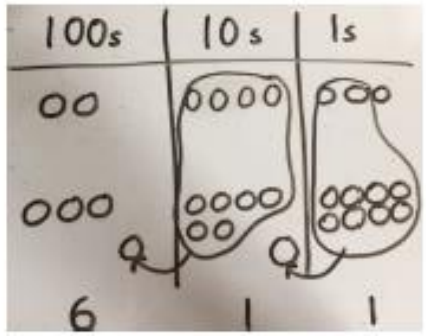
Add fractions with the same denominator within one whole e.g 5/7 + 1/7 = 6/7

Mathematical Vocabulary

Pupils should read, spell and pronounce mathematical vocabulary correctly.

add, addition, more, plus, altogether, total, equal to, equals, more, and, make, sum, double, count on, increase, column, tens, ones, most, least, number line, partition, tens boundary, hundreds boundary, vertical, exchange, expanded, compact.

YEAR THREE – Addition

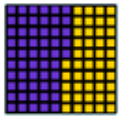
Concrete	Pictorial	Abstract
Use of place value counters to add HTO + TO, HTO + HTO etc. When there are 10 ones in the 1s column- we exchange for 1 ten, when there are 10 tens in the 10s column- we exchange for 1 hundred. 	Chidren to represent the counters in a place value chart, circling when they make an exchange. 	$\begin{array}{r} 243 \\ +368 \\ \hline 611 \\ \hline 1\ 1 \end{array}$

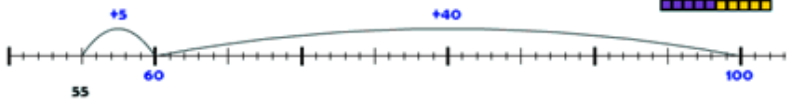
Subtract with numbers up to 3-digits

Children will continue to practise a **wide range of mental subtraction strategies** and use **empty number lines,** including **bridging through 100,** with increasingly large numbers. They use **compensation,** where appropriate, and begin to use more **formal written methods**.

Number bonds

Children will develop their use of number bonds to include the **'story' of 100**: **100 – 55 = 45**





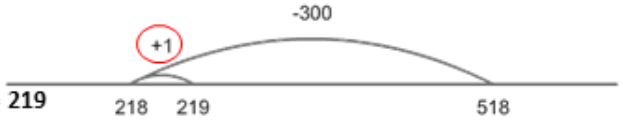
Subtracting near multiples of 10/100

Children continue to use the **empty number line.** where appropriate, to support mental subtraction of numbers that are close multiples of 10/100 (ending in 1, 2, 8 or 9) e.g.

518 - 299

Becomes...

518 - 300 (+ 1) = 219



Counting back or counting on?

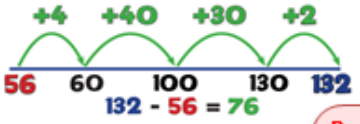
Children are encouraged to make decisions about the most appropriate mental method to use, based on the numbers involved e.g.

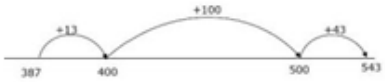
Counting up for:
402 – 387

Counting back *(taking away using partitioning)* **for:**
401 - 22

Bridging multiples of 10/100

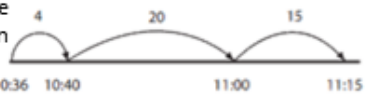
When working mentally with larger numbers, it can help to count in steps, bridging multiples of 10/100.





Bridging 60 (time intervals)

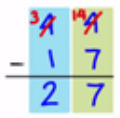
Children bridge 60 to calculate time intervals e.g. Counting on from 36 to 40, then to 60, to give a total of 24 minutes.



Bar modelling

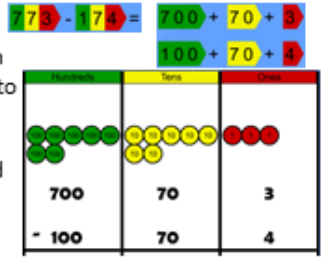
Children use bar modelling to help them solve one-step word problems and begin to use it for two-step problems e.g.

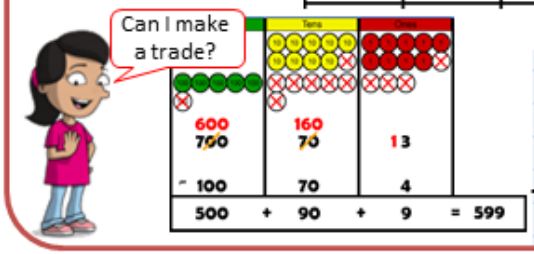
On Saturday, Jack sold 44 ℓ of lemonade.
He sold 17 ℓ less of orange juice.
How many litres of orange juice did Jack sell?



Place value counters

Place value counters are used to model subtraction in columns, working right to left, initially without decomposition, by 'taking away'. The written method is then be recorded alongside the model.

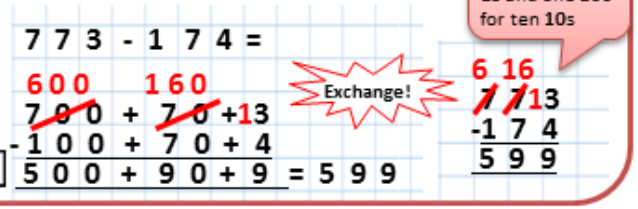




Formal Written Method

Introducing the formal method alongside expanded addition, ensures children see it as a more streamlined version, as opposed to a completely new approach.

'Exchanging' one **10** for ten **1**s and one **100** for ten **10**s



Subtraction Y3

Key skills:

- Count from **0** in multiples of **4, 8, 50** and **100** and steps of **1/10**
  - (children will now be using multiples of **2, 3, 4, 5, 8, 50** and **100)**
- Find **10** or **100 more** than a given number.
- Recognise the **place value** of each digit in a **3-digit number** (hundreds, tens and ones)
- Compare and order numbers **up to 1000**
- Read and write numbers **up to 1000** in numerals and in words.
- Subtract numbers **mentally**, including:
  - a 3-digit number and ones (e.g. 127 - 8) **HTU - U**
  - a 3-digit number and tens (e.g. 223 - 40) **HTU - T**
  - a 3-digit number and hundreds (e.g. 723 - 200) **HTU - H**
- Subtract numbers with **up to 3-digits** using **formal written methods** of column addition. (e.g. 723 - 217) **HTU - HTU**
- **Estimate** the answer to a calculation and use **inverse** operations to check answers.
- **Solve number and practical problems**, including **missing number problems**, using number facts, place value, and more complex addition.

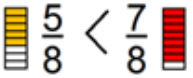
Fractions

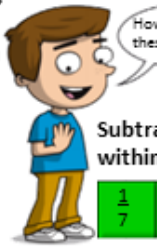
Children will learn to:

- **count up** and **down in tenths;**
- Recognise and use **fractions as numbers** on a **number line**
- **deduce relationships** between fractions, such as **size** and **equivalence**;

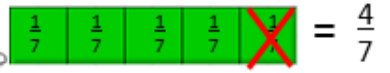
Children will go beyond the [0, 1] interval, including relating this to measure.

Compare and order **unit fractions** and fractions with the **same denominators**.





Subtract fractions with the same denominator **within one whole** e.g. **5/7 - 1/7 = 4/7**

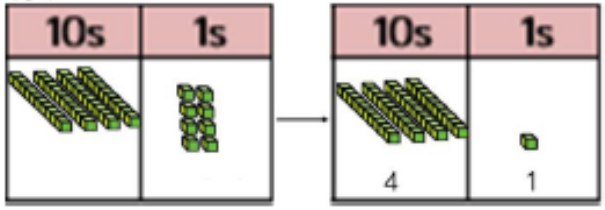
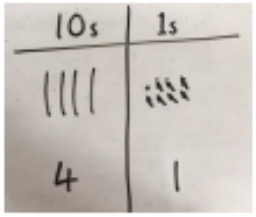
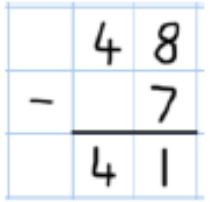
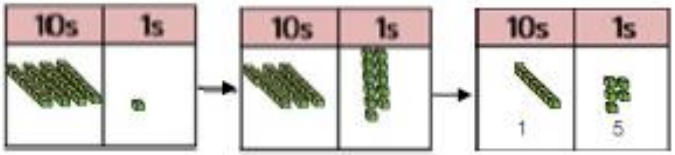
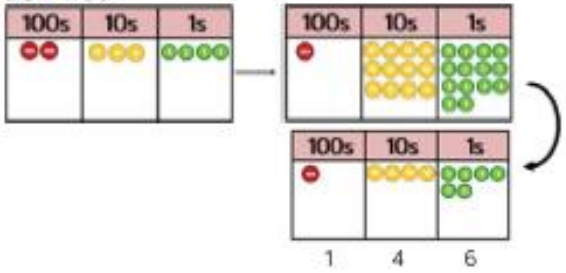
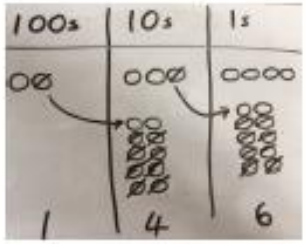
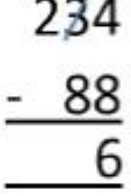


Mathematical Vocabulary

Pupils should read, spell and pronounce mathematical vocabulary correctly.

Take, take away, minus, subtract, subtraction, difference between, less than, more, total, equal to, equals, more, count on, decrease, least, how many left?, how much less is...?, column subtraction, inverse, hundreds, tens, ones, most, least, number line, partition, tens boundary, hundreds boundary, vertical, column, exchange, expanded, compact.

YEAR THREE - Subtraction

Concrete	Pictorial	Abstract
Column method using base 10. 48-7 	Children to represent the base 10 pictorially. 	Column method or children could count back 7. 
Column method using base 10 and having to exchange. 41 – 26 	Represent the base 10 pictorially, remembering to show the exchange. 	Formal column method. Children must understand that when they have exchanged the 10 they still have 41 because 41 = 30 + 11. 
Column method using place value counters. 234 – 88 	Represent the place value counters pictorially; remembering to show what has been exchanged. 	Formal colum method. Children must understand what has happened when they have crossed out digits. 

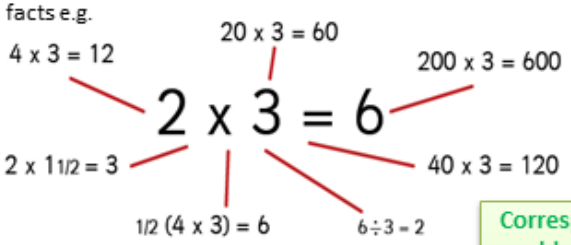
Multiplication Y3

Multiply 2 digits by a single-digit number

Children continue to practise their **mental recall of multiplication tables** when they are calculating mathematical statements in order to improve fluency. They develop **efficient mental methods** as well as **reliable written methods** for multiplication, starting with calculations of **two-digit numbers by one-digit numbers** and progressing to the **formal written methods of short multiplication**. They solve **simple multiplication problems in real life contexts**, including **measuring**, **scaling** and **correspondence** problems, deciding which operations to use and why.

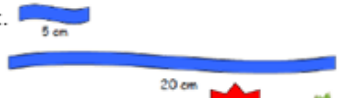
Using facts

Children work on achieving instant recall of 3, 4 and 8 multiplication tables and use these to derive associated facts e.g.



Measuring and scaling

They solve problems involving measuring and scaling for example, 'four times as high,' eight times as long etc.



Strawberries cost 80p for 200g. How much would you get for 40p?

Commutativity and associativity

Children develop mental strategies using these concepts, for example

4 × 12 × 5 =

4 × 5 × 12 =

20 × 12 = 240

Correspondence problems

Children solve problems in which m objects are connected to n objects : 12 sweets shared equally between 4 children; 4 cakes shared equally between 8 children.

3 hats and 4 coats, how many different outfits? 3 x 4 = 12

Grid Method

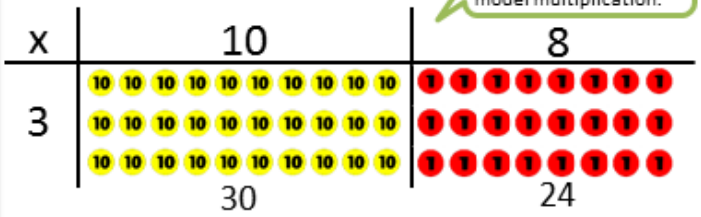
Children are first introduced to the grid method for multiplying a 2-digit number by a single digit using the visual image of an **array**. This develops into an **area** model.

Partitioning

When multiplying a two-digit number or greater, it is partitioned into tens and ones.

e.g. 18 x 3

Children use **place value counters** and charts to model multiplication.



Grid method without counters...

x	10	8
3	30	24

30 + 24 = 54

Exchange

Counters can be exchanged when adding the products, by exchanging one 10 for ten 1s.

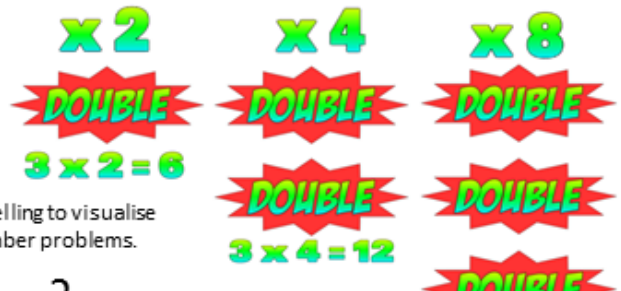
Key skills:

- Count from **0** in multiples of **4, 8, 50** and **100** and steps of **1/10** (children will now be using multiples of **2, 3, 4, 5, 8, 50** and **100)**
- Recall and use multiplication and division facts for the **3, 4** and **8** multiplication tables.
- Write and calculate mathematical statements for multiplication and division using the multiplication tables that they know, including for two-digit numbers times one-digit numbers **TU x U**, using mental and progressing to formal written methods.
- Solve problems, including **missing number problems**, involving multiplication and division, including positive integer **scaling problems** and **correspondence problems** in which n objects are connected to m objects.

Doubling

Through **doubling**, children **connect** the **2, 4** and **8 multiplication tables**, for example,

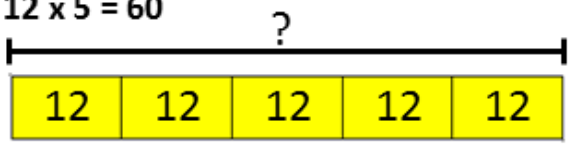
3 x 2 is double 3
3 x 4 double 3 then double again
3 x 8 double, double and double again



Bar modelling

Children use bar modelling to visualise word and missing number problems.

12 x 5 = 60

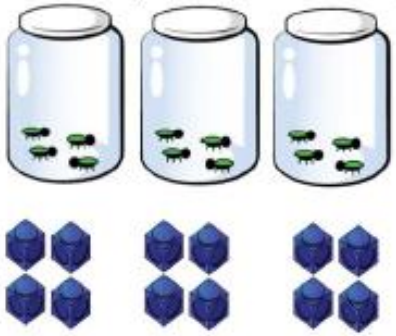
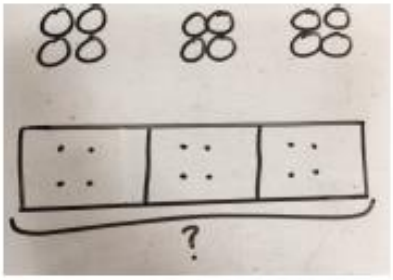
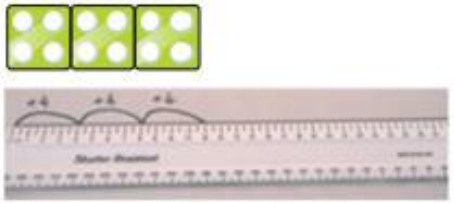
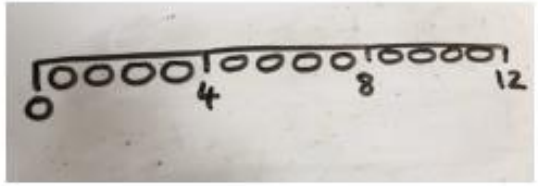
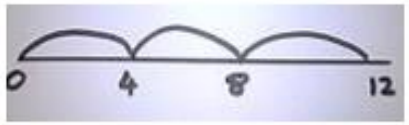
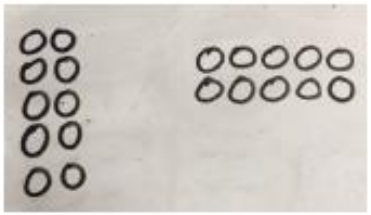


Mathematical Vocabulary

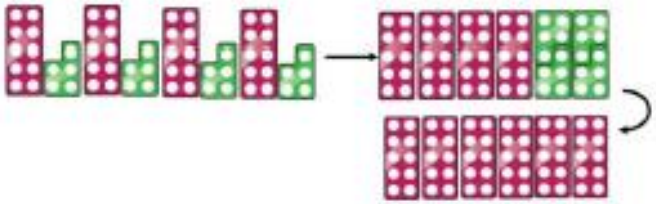
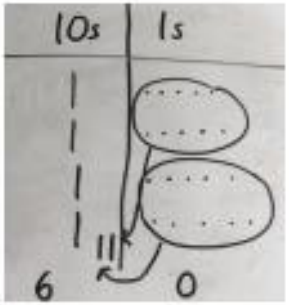
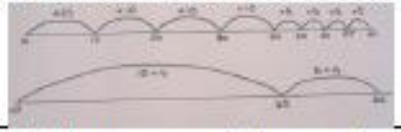
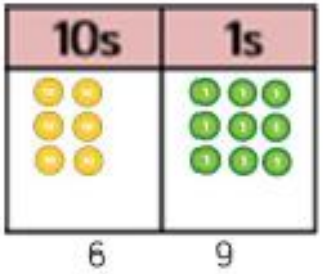
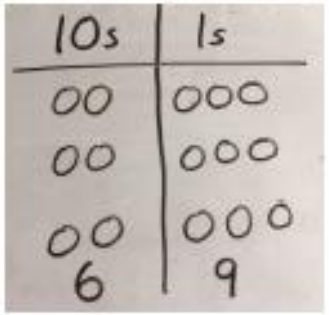
Pupils should read, spell and pronounce mathematical vocabulary correctly.

group, groups of, times, array, altogether, column, row, sets, multiply, count, multiplied by, grid method, multiple, product, inverse, repeated addition, column, row, sets of, equal groups, times, once, twice, three times..., partition, grid method, multiple, value, square, factor, decimal, BIDMAS

YEAR THREE - Multiplication

Concrete	Pictorial	Abstract
Repeated grouping/repeated addition 3 × 4 4 + 4 + 4 There are 3 equal groups, with 4 in each group. 	Children to represent the practical resources in a picture and use a bar model. 	3 × 4 = 12 4 + 4 + 4 = 12
Number lines to show repeated groups- 3 × 4  Cuisenaire rods can be used too.	Represent this pictorially alongside a number line e.g.: 	Abstract number line showing three jumps of four. 3 × 4 = 12 
Use arrays to illustrate commutativity counters and other objects can also be used. 2 × 5 = 5 × 2  2 lots of 5 5 lots of 2	Children to represent the arrays pictorially. 	Children to be able to use an array to write a range of calculations e.g. 10 = 2 × 5 5 × 2 = 10 2 + 2 + 2 + 2 + 2 = 10 10 = 5 + 5

YEAR THREE - Multiplication

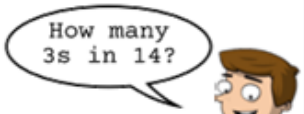
Partition to multiply using Numicon, base 10 or Cuisenaire rods. 4 × 15 	Children to represent the concrete manipulatives pictorially. 	Children to be encouraged to show the steps they have taken. 4 × 15 10 5 10 × 4 = 40 5 × 4 = 20 40 + 20 = 60 A number line can also be used 
Formal column method with place value counters (base 10 can also be used.) 3 × 23 	Children to represent the counters pictorially. 	Children to record what it is they are doing to show understanding. 3 × 23 20 3 3 × 20 = 60 3 × 3 = 9 60 + 9 = 69 23 × 3 69

Division Y3

Divide 2 digit numbers by a single digit

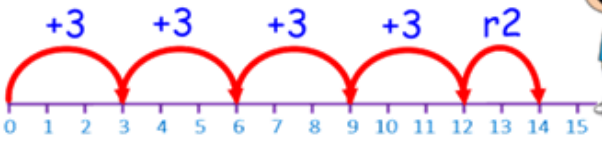
Children continue to practise their **mental recall of multiplication tables** when they are calculating mathematical statements in order to improve fluency. Children develop **efficient mental methods** as well as **reliable written methods** for division, starting with calculations of **two-digit numbers by one-digit numbers** and progressing to the **formal written methods of short division**. They solve division problems in real life contexts, including measuring, scaling and correspondence problems.

Introducing remainders



Division on a number line

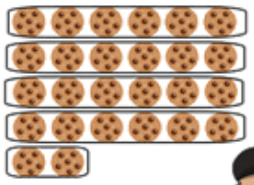
Children continue to solve division problems by **grouping on a number line** from zero. At this stage they are introduced to the concept of remainders through practical activities.



Division as an array

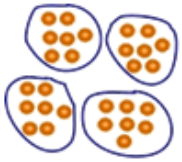
Children continue to solve division problems by **grouping on a number line** from zero. At this stage they are introduced to the concept of remainders through practical activities.

26 ÷ 6 = 4 r 2

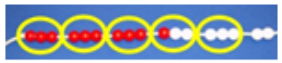


Grouping

Understanding of division is developed through grouping objects.



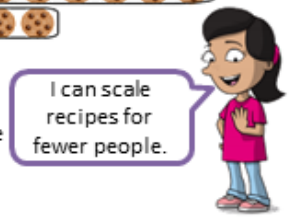
30 ÷ 7 = 4 r2



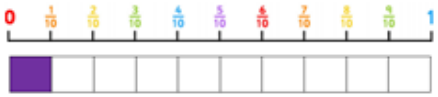
17 ÷ 3 = 5 r2

Scaling problems

For example, half as high, a quarter of the length.



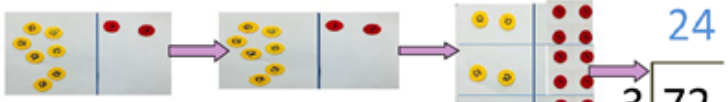
Pupils connect tenths to division by 10.



Place value counters

By working with place value counters, children gain experience of exchanging numbers (repartitioning). This illustrates the connection between multiplication and division.

One of the 10 counters has been exchanged for ten 1's.

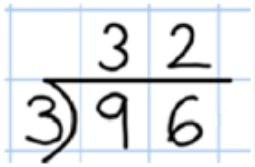


Using known facts

Children use multiplication and division facts, e.g. using
3 × 2 = 6, 6 ÷ 3 = 2
and 2 = 6 ÷ 3
to derive related facts e.g.
30 × 2 = 60, 60 ÷ 3 = 20
and 20 = 60 ÷ 3

Formal Written Method

Once children are secure with division as grouping, demonstrated by using number lines and arrays, they progress to using a more formal written method of short division to divide a 2-digit number by a single digit. They use known number facts **9 ÷ 3 = 3** to derive **90 ÷ 3 = 30**, asking, 'How many 3s in 9?'



When introduced to the formal written method, each digit in the question must be a multiple of the divisor, so there will not be a 'carried' digit.

Key skills:

- Count from **0** in multiples of **4, 8, 50** and **100** and steps of **1/10**
  - (children will now be using multiples of **2, 3, 4, 5, 8, 50** and **100)**
- Recall and use multiplication and division facts for the **3, 4** and **8** multiplication tables.
- Write and calculate mathematical statements for multiplication and division using the multiplication tables that they know, including for two-digit numbers times one-digit numbers **TU x U**, using mental and progressing to formal written methods.
- Solve problems, including **missing number problems**, involving multiplication and division, including positive integer **scaling problems** and **correspondence problems** in which n objects are connected to m objects.

Linking to fractions

Children continue to recognise fractions in the context of parts of a whole, numbers, measurements, a shape, and unit fractions as a division of a quantity.

To show that 3 ÷ 4 = $\frac{3}{4}$

3 pizzas are divided among 4 friends.

So, each of the friends gets 1/4 of each of the 3 pizzas

Bar modelling

The bar model illustrates the link between division and fractions and is used to support children in visualising one-step word problems.

If Teresa shared 20 apples between 5 people, how many apples would each person get?

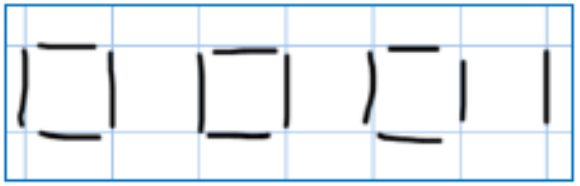
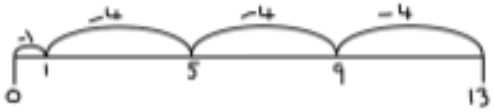
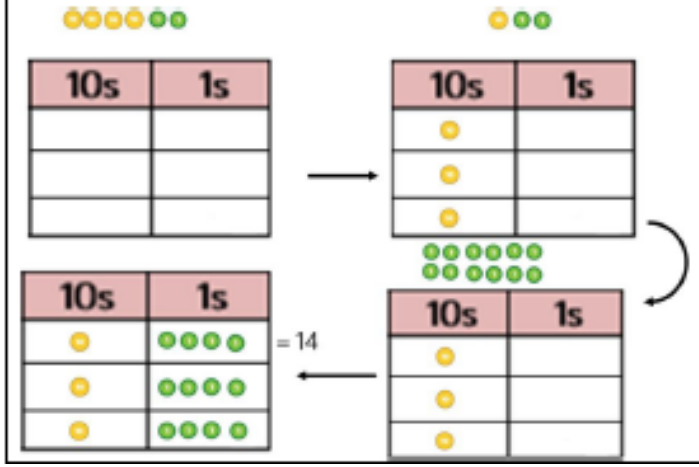
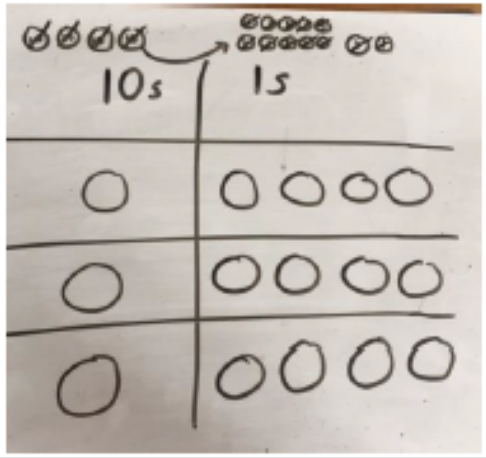
20 ÷ 5 = $\frac{20}{5}$

Mathematical Vocabulary

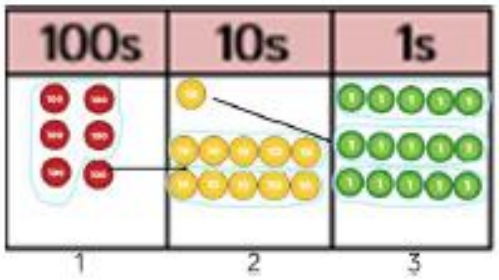
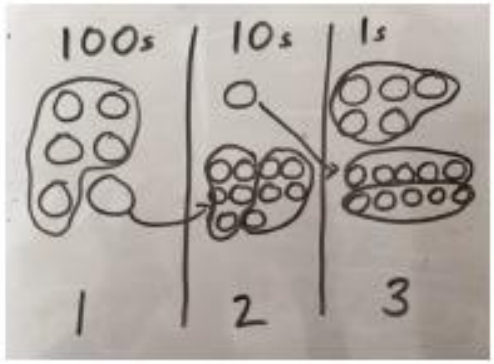
Pupils should read, spell and pronounce mathematical vocabulary correctly.

Share, share between, share equally, group, groups of, lots of, array, divide, divided by, divided into, division, left, left over, grid method, multiple, product, inverse, short division, 'carry', remainder

YEAR THREE - Division

Concrete	Pictorial	Abstract
2d ÷ 1d with remainders using lollipop sticks. Cuisenaire rods, above a ruler can also be used. 13 ÷ 4 Use of lollipop sticks to form wholes- squares are made because we are dividing by 4.  There are 3 whole squares, with 1 left over.	Children to represent the lollipop sticks pictorially.  There are 3 whole squares, with 1 left over.	13 ÷ 4 = 3 remainder 1 Children should be encouraged to use their times table facts; they could also represent repeated addition on a number line. '3 groups of 4, with 1 left over' 
Sharing using place value counters. 42 ÷ 3 = 14 	Children to represent the place value counters pictorially. 	Children to be able to make sense of the place value counters and write calculations to show the process. 42 ÷ 3 42 = 30 + 12 30 ÷ 3 = 10 12 ÷ 3 = 4 10 + 4 = 14

YEAR THREE - Division

Short division using place value counters to group. 615 ÷ 5  1. Make 615 with place value counters. 2. How many groups of 5 hundreds can you make with 6 hundred counters? 3. Exchange 1 hundred for 10 tens. 4. How many groups of 5 tens can you make with 11 ten counters? 5. Exchange 1 ten for 10 ones. 6. How many groups of 5 ones can you make with 15 ones?	Represent the place value counters pictorially.  	Children to the calculation using the short division scaffold. $5\overline{)615}$ 123 $^{1}6\,^{1}1\,5$

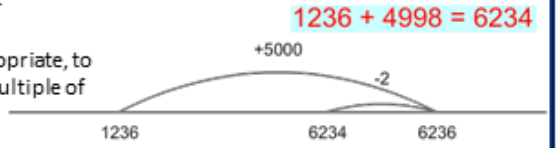
Addition Y4

Add numbers with up to 4-digits

Children will continue to practise a wide range of **mental addition strategies** and more **formal written column methods** with numbers up to 4-digits, including **compensation** where appropriate. They will extend the use of the **number line** to connect fractions, numbers and measures.

Near multiples

The **empty number line** continues to be used where appropriate, to support **mental addition** of numbers that are close to a multiple of 100/1000 e.g. **1236 + 4998 = 1263 + 5000 - 2**



1
0.8 + 0.2
0.6 + 0.4
0.5 + 0.5
0.9 + 0.1

Number bonds

Children develop their use of number bonds to include the story of 1, using decimals to 1dp. They use number bonds to reorder calculations.

Partitioning and recombining

Children will apply partitioning in a variety of ways, related to place value, using varied and increasingly complex problems, to calculate mentally. They will continue to practise adding fractions with the same denominator, beyond one whole.

Using near doubles

Children use near doubles e.g. using

800 + 800 = 1600

To work out...

820 + 814

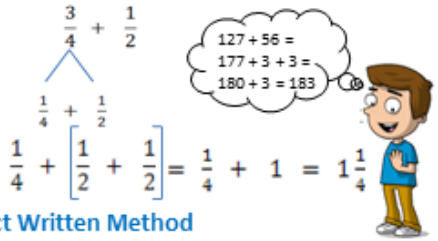
= 800 + 800 + 20 + 14

=1600 + 20 + 14 = 1634

£1.60 + 80p

(£1.60 + 40p) + 40p

£2 + 40p = £2.40



Counting on

Children partition the smallest number and count on:

6.5 + 3.7

= 6.5 + 3 + 0.5 + 0.2

= 9.5 + 0.5 + 0.2

= 10 + 0.2 = 10.2

They use number facts to solve missing number/ digit problems:

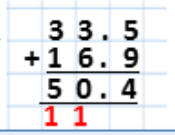
456 + □ = 710

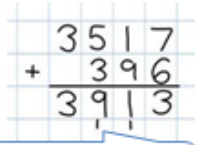
1□7 + 6□ = 200

60 + 99 + □ = 340

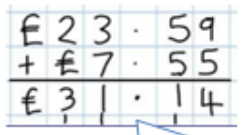
Compact Written Method

Children progress to using the compact method for addition of numbers with up to 4-digits. In the context of money and measures calculations are extended to numbers with up to 2 decimal places, and adding several numbers with different amounts of digits.





'Carried' digits are written below.



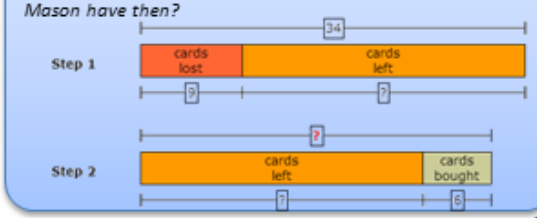
Children learn that decimal points must line up.

Place value counters and charts reinforce understanding of place value and exchange.

Bar modelling

Children use bar models to help them visualise and solve two-step word problems e.g.

Mason had 34 superhero trading cards. He lost 9 cards at the park. He bought 6 cards at the store. How many cards did Mason have then?



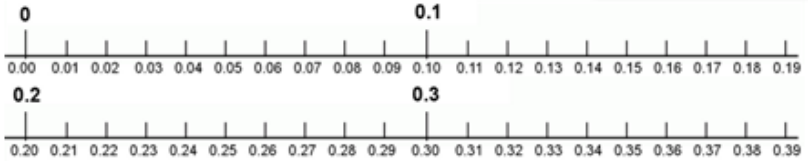
Key skills:

- Count in multiples of **6, 7, 9, 25** and **1000** and steps of **1/100** (children will be using multiples of **2, 3, 4, 5, 6, 7, 8, 9, 25, 50** and **100)**
- Find **1000 more** than a given number.
- Count **back through zero** to include **negative** numbers
- Recognise the **place value** of each digit in a **4-digit number** (thousands, hundreds, tens and ones)
- Compare and order numbers **beyond 1000**
- Add numbers with **up to 4-digits** using **formal written methods** of column addition, where appropriate. **(ThHTU + ThHTU)**
- **Estimate** and use **inverse** operations to **check** answers.
- Solve addition, **two-step problems in contexts**, deciding which operations and methods to use and why.

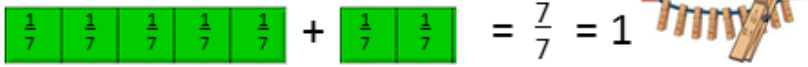
Fractions

- Children should count up and down in **hundredths (fractions and decimals)**, recognising that hundredths arise from dividing one by 100 and tenths by ten.
- **Solve problems** involving increasingly harder fractions to calculate quantities, including non-unit fractions where the **answer is a whole number**.

Children learn to recognise and write decimal equivalents of any number of tenths or hundredths



- Add fractions with the same **denominator**.

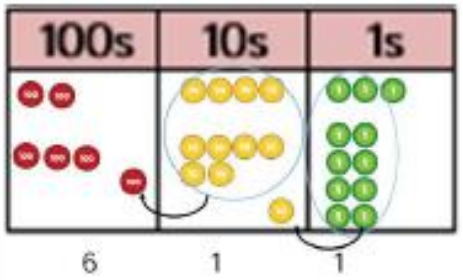
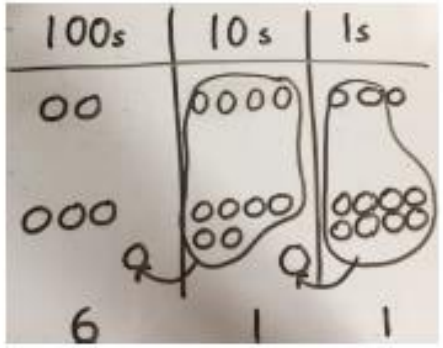


Mathematical Vocabulary

Pupils should read, spell and pronounce mathematical vocabulary correctly.

add, addition, more, plus, altogether, total, equal to, equals, more, and, make, sum, double, count on, increase, column, tens, ones, most, least, number line, partition, recombine tens boundary, hundreds boundary, vertical, exchange, expanded, compact perimeter.

YEAR FOUR - Addition

Concrete	Pictorial	Abstract
Use of place value counters to add HTO + TO, HTO + HTO etc. When there are 10 ones in the 1s column- we exchange for 1 ten, when there are 10 tens in the 10s column- we exchange for 1 hundred. 	Chidren to represent the counters in a place value chart, circling when they make an exchange. 	243 +368 611 1 1

Subtract numbers with up to 4-digits

Children will continue to practise a wide range of **mental subtraction strategies** and more **formal written column methods** with numbers up to **4-digits**, using decomposition where appropriate. They will extend the use of the **number line** to connect fractions, numbers and measures.

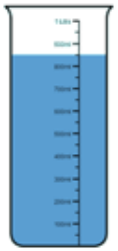
Partitioning and recombining

Children will apply partitioning in a variety of ways, related to place value, using varied and increasingly complex problems, to calculate mentally, including **bridging through 60 to calculate time intervals.**

Counting back

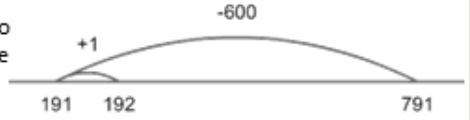
Children partition the smallest number and count back:

6.5cm - 3.7cm
= 6.5 - 3 - 0.5 - 0.2
= 3.5 - 0.5 - 0.2
= 3 - 0.2 = 2.8cm



Counting on and Bridging through multiples of 10

e.g. 850ml – 268 ml
= 268 + 32 + 500 + 50

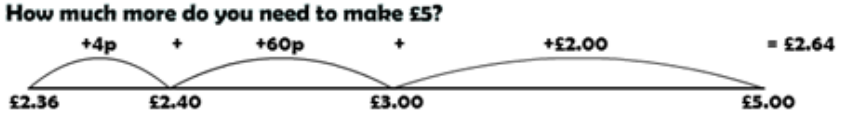


Near multiples

The **empty number line** continues to be used where appropriate, to support **mental subtraction** of numbers that are close to a multiple of 100/1000 e.g. **791 – 599 = 791 – 600 + 1 = 192**

Counting on

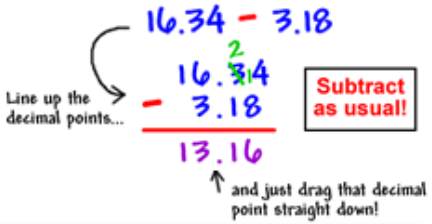
Children will count on to find the difference.



They use number facts to solve missing number/ digit problems:
□ - 710 = 456 108 - 6□ = 42 340 - □ = 99

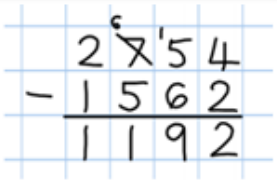
In the context of money and measures

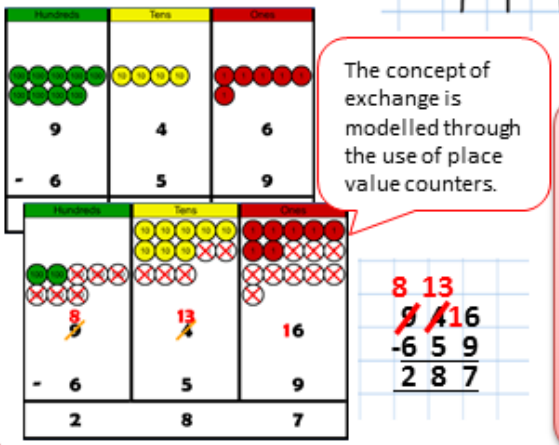
calculations are extended to numbers with **up to 2 decimal places** with same number of decimal places..



Compact Written Method

Children progress to using the compact method for subtraction of numbers with **up to 4-digits**.



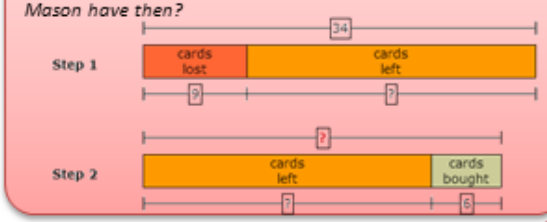


The concept of exchange is modelled through the use of place value counters.

Bar modelling

Children use bar models to help them visualise and solve two-step word problems e.g.

Mason had 34 superhero trading cards. He lost 9 cards at the park. He bought 6 cards at the store. How many cards did Mason have then?



Subtraction Y4

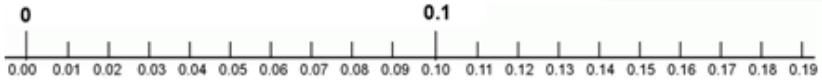
Key skills:

- Count in multiples of **6, 7, 9, 25** and **1000** and steps of **1/100** (children will be using multiples of **2, 3, 4, 5, 6, 7, 8, 9, 25, 50** and **100)**
- Find **1000 less** than a given number.
- Count **back through zero** to include **negative** numbers
- Recognise the **place value** of each digit in a **4-digit number** (thousands, hundreds, tens and ones)
- Compare and order numbers **beyond 1000**
- Subtract numbers with **up to 4-digits** using **formal written methods** of column subtraction, where appropriate. **(ThHTU - ThHTU)**
- **Estimate** and use **inverse** operations to **check** answers.
- Solve subtraction, **two-step problems in contexts**, deciding which operations and methods to use and why.

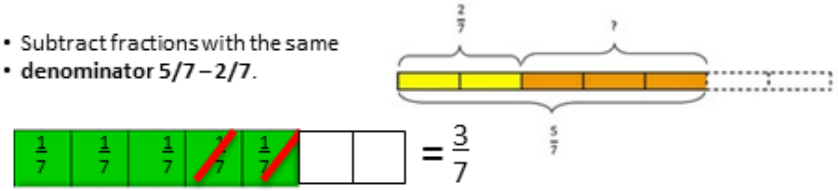
Fractions

- Children should count up and down in **hundredths (fractions and decimals)**, recognising that hundredths arise from dividing one by 100 and tenths by ten.
- **Solve problems** involving increasingly harder fractions to calculate quantities, including non-unit fractions where the **answer is a whole number**.

Children learn to recognise and write decimal equivalents of any number of tenths or hundredths



- Subtract fractions with the same
- **denominator 5/7 – 2/7.**

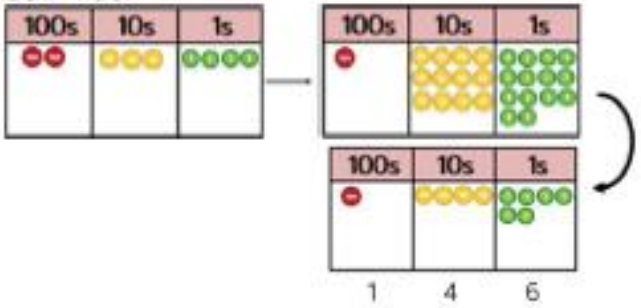
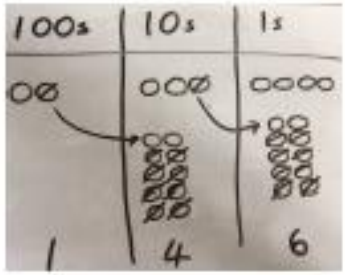


Mathematical Vocabulary

Pupils should read, spell and pronounce mathematical vocabulary correctly.

Take, take away, minus, subtract, subtraction, difference between, less than, more, total, equal to, equals, more, count on, decrease, least, how many left?, how much less is...?, column subtraction, inverse, hundreds, tens, ones, most, least, number line, partition, tens boundary, hundreds boundary, vertical, column, exchange, expanded, compact.

YEAR FOUR - Subtraction

Concrete	Pictorial	Abstract
Column method using place value counters. 234 – 88 	Represent the place value counters pictorially; remembering to show what has been exchanged. 	Formal colum method. Children must understand what has happened when they have crossed out digits. $\begin{array}{r} {}^{2}\not{2}\,{}^{1}\not{3}4 \\ -\ 88 \\ \hline 6 \end{array}$

Multiplication Y4

Multiply two- and three-digit numbers by a single digit, *using all multiplication tables up to 12x12*

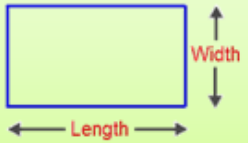
Children continue to practise **recalling** and **using multiplication tables** and related division facts to aid fluency. They **practise mental methods** and extend this to **three-digit numbers** to derive facts, (for example 600 ÷ 3 = 200 can be derived from 2 x 3 = 6). They practise to become fluent in the **formal written method** of **short multiplication** and combine their knowledge of number facts and rules of arithmetic to solve mental and written calculations for example, 2 x 6 x 5 = 10 x 6 = 60. They write statements about the equality of expressions (for example, use the distributive law 39 × 7 = 30 × 7 + 9 × 7 and associative law (2 × 3) × 4 = 2 × (3 × 4)).

Counting and times tables

Children continue to count on regularly, now including multiples of **6, 7, 9, 25** and **100** and steps of **1/000.** They work on achieving instant recall of all multiplication tables to 12 x 12.

Area

The connection is made between multiplication as an array and as the area of a rectangle.



Approximate/estimate

Children are encouraged to **approximate/estimate** before undertaking a calculation.

e.g.
346 x 9 is approximately ...
350 x 10 = 3500

Scaling problems

Children develop their understanding of multiplication as scaling by scaling up e.g. ***How tall would a 25cm sunflower be if it was 6 times taller?***

Grid method

Children deepen their understanding of multiplication using the grid method. When multiplying a two-digit number or greater, children must understand how to split the number into H, T and U.

x	100	20	3
5	500	100	15

$$\begin{array}{r} 500 \\ 100 \\ +\quad 15 \\ \hline 615 \end{array}$$

Short multiplication method

Once children are confident with exchange for addition, they progress to using the formal written method for multiplication. Children are encouraged to identify the connections between the **formal** and **grid methods**, considering what is the **same/ different**.

step 1

$$\begin{array}{r} 123 \\ \times\quad 5 \\ \hline 5 \\ \hline 1 \end{array}$$

step 2

$$\begin{array}{r} 123 \\ \times\quad 5 \\ \hline 15 \\ \hline 11 \end{array}$$

step 3

$$\begin{array}{r} 123 \\ \times\quad 5 \\ \hline 615 \\ \hline 11 \end{array}$$

Connections are highlighted between the grid method and the formal layout when introducing this method.

'Carried' digits are written below.

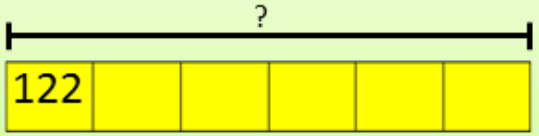
Key skills:

- Count in multiples of **6, 7, 9, 25** and **1000** and steps of **1/100**
- (children will be using multiples of **2, 3, 4, 5, 6, 7, 8, 9, 25, 50** and **100)**
- Recall multiplication and division facts for multiplication tables up to 12 × 12
- Use place value, known and derived facts to multiply and divide mentally, including:
  - multiplying by 0 and 1;
  - dividing by 1;
  - multiplying together three numbers
- Recognise and use **factor pairs** and **commutativity** in mental calculations
- to multiply **two digit** and **three digit** numbers by **one digit** number using the **formal written layout**
- to be able to solve problems involving multiplying and adding, including using the distributive law to multiply two digit numbers by one digit, **integer scaling** problems and **harder correspondence** problems such as n objects are connected to m objects (see example in Y3)

Bar modelling

Children use bar models to help them visualise and solve multiplication and division problems.

The Sidewinder bike costs £122. The Rocket bike costs 6 times as much. How much does the rocket bike cost?



Algebra

Children investigate missing number problems, e.g. Put the numbers 7.2, 8, 0.9 in the boxes to make the number sentences correct.

□ = □ x □

□ = □ ÷ □

Mathematical Vocabulary

Pupils should read, spell and pronounce mathematical vocabulary correctly.

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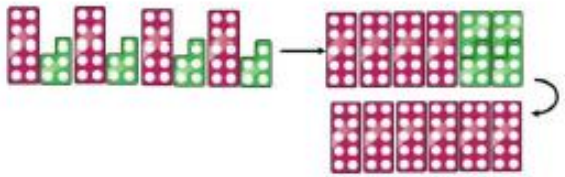
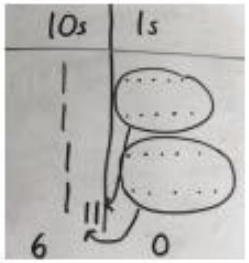
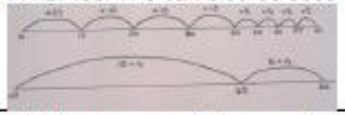
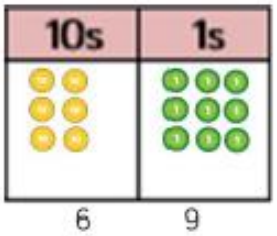
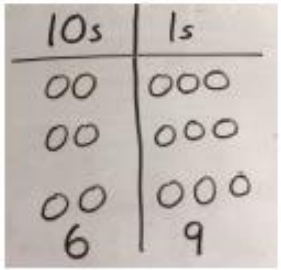
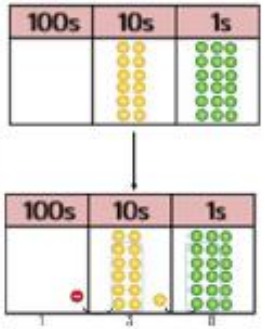
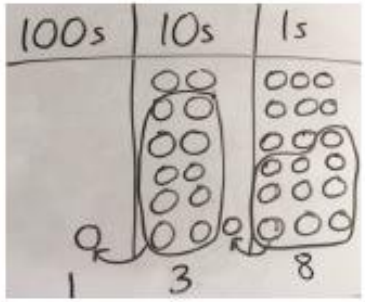
group, groups of, times, array, altogether, column, row, sets, multiply, count, multiplied by, grid method, multiple, product, inverse, repeated addition, column, row, sets of, equal groups, times, once, twice, three times..., partition, grid method, multiple, value, square, factor, integer, decimal, short, decimal, square number BIDMAS

.

YEAR FOUR - Multiplication

Concrete	Pictorial	Abstract
Partition to multiply using Numicon, base 10 or Cuisenaire rods. 4 × 15 	Children to represent the concrete manipulatives pictorially. 	Children to be encouraged to show the steps they have taken. 4 × 15 10 5 10 × 4 = 40 5 × 4 = 20 40 + 20 = 60 A number line can also be used 
Formal column method with place value counters (base 10 can also be used.) 3 × 23 	Children to represent the counters pictorially. 	Children to record what it is they are doing to show understanding. 3 × 23 20 3 3 × 20 = 60 3 × 3 = 9 60 + 9 = 69 23 × 3 69
Formal column method with place value counters. 6 x 23 	Children to represent the counters/base 10, pictorially e.g. the image below. 	Formal written method 6 x 23 = 23 × 6 138 1 1

Division Y4

Divide up to 3-digit numbers by a single digit

Children continue to practise recalling and using multiplication tables and related division facts to aid fluency. They practise mental methods and extend this to three-digit numbers to derive facts, (for example 600 ÷ 3 = 200 can be derived from 2 x 3 = 6). They practise to become fluent in the **formal written method** of **short division** with **exact answers**. They combine their knowledge of number facts and rules of arithmetic to solve mental and written calculations for example, 2 x 6 x 5 = 10 x 6 = 60.

Mental Strategies

Children calculate mentally using the 'build up' method (where they 'build up' multiples to find the total amount being divided.

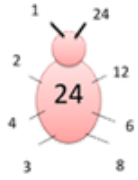
832 ÷ 8 = ?

100 x 8 = 800
4 x 8 = 32

So, *104* x 8 = 832
and 832 ÷ 8 = *104*

Factor pairs

Children learn to recognise and use factor pairs in mental calculations.



Using known facts

Children divide multiples of 100 by 1-digit numbers using division facts:

600 ÷ 3 = 200
can be derived from
2 x 3 = 6.

Inverse relationships

Children continue to investigate fact families and missing signs and missing number/ digit problems with increasingly complex numbers to strengthen understanding of the inverse relationship between multiplication and division and the equality of expressions.

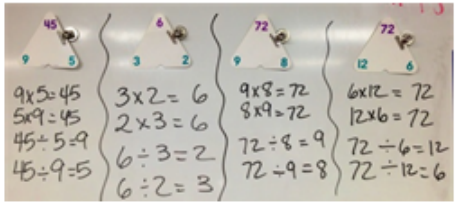
8 = 32 ÷ ? 7 x ? = 28 ÷ 2

Partitioning

Partitioning is used to find halves of even numbers and to halve amounts of money e.g.

half of £52·40 is £26·20

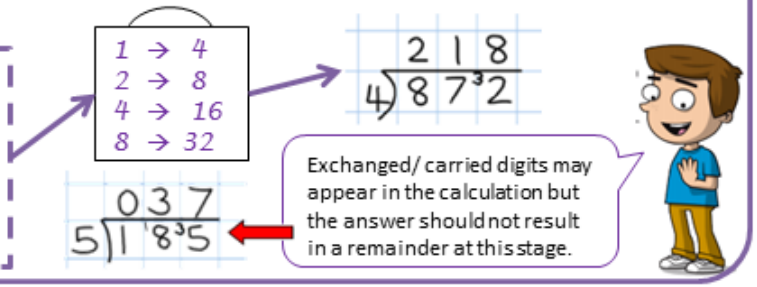
Fact family triangles:



Written Method

The short division algorithm is practised, dividing **3 digit numbers** by a **single digit**. Real-life contexts are used routinely to help children gain full understanding and develop their ability to recognise the place of division and how to apply it to problems..

Jottings
Jottings (multiples) are used to support both mental and written methods. Children should be taught to use them flexibly, writing as much or as little down as required.



Mathematical Vocabulary

Pupils should read, spell and pronounce mathematical vocabulary correctly.

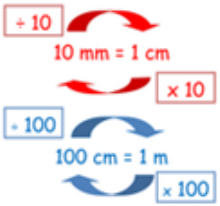
share, share between, share equally, group, groups of, lots of, array, divide, divided by, divided into, division, left, left over, grid method, multiple, product, inverse, short division, 'carry', remainder, factor, quotient, divisor

Key skills:

- Count in multiples of **6, 7, 9, 25** and **1000** and steps of **1/100**
- (children will be using multiples of **2, 3, 4, 5, 6, 7, 8, 9, 25, 50** and **100)**
- Recall multiplication and division facts for multiplication tables up to 12 × 12
- Use place value, known and derived facts to multiply and divide mentally, including:
  - multiplying by 0 and 1;
  - dividing by 1;
  - multiplying together three numbers
- Recognise and use factor pairs in mental calculations
- Solve problems involving multiplying and dividing, including 2-step problems in context (choosing the appropriate operation), integer scaling problems and harder correspondence problems such as n objects are connected to m objects, (for example, the number of choices of a meal on a menu, or three cakes shared equally between 10 children.

Measurement

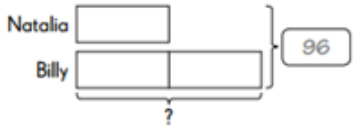
Children investigate the effect of multiplying and dividing a one- or two-digit number by 10 and 100, identifying the value of the digits in the answer as ones, tenths and hundredths and use this knowledge to convert between units of measure, for example converting m to cm etc.



Bar modelling

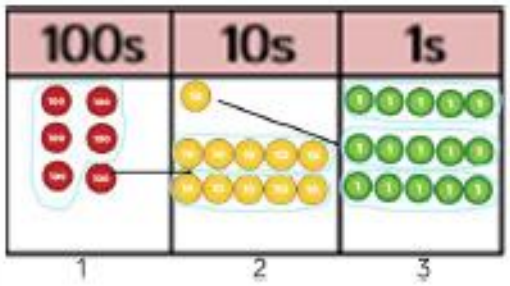
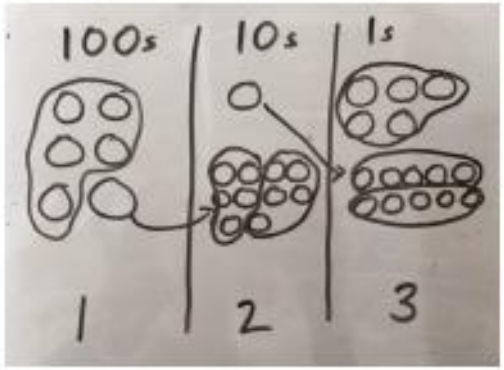
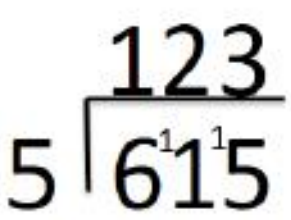
Children use bar models to help them visualise and solve two-step word problems involving division.

Billy has twice as many stamps as Natalia.
They have 96 stamps altogether.
How many stamps does Billy have?



3 units → 96
1 unit → 96 ÷ 3 = 32
2 units → 2 × 32 = 64
Billy has 64 stamps.

YEAR FOUR - Division

Short division using place value counters to group. 615 ÷ 5  1. Make 615 with place value counters. 2. How many groups of 5 hundreds can you make with 6 hundred counters? 3. Exchange 1 hundred for 10 tens. 4. How many groups of 5 tens can you make with 11 ten counters? 5. Exchange 1 ten for 10 ones. 6. How many groups of 5 ones can you make with 15 ones?	Represent the place value counters pictorially.  	Children to the calculation using the short division scaffold. 

Addition Y5

Add numbers with more than 4-digits

Children practise using the formal written methods of **columnar addition** with increasingly large numbers to aid fluency (e.g. 10 162 – 2 300 = 12 462). Place value counters and number lines are used to consolidate and secure understanding. They practise **adding fractions** to become fluent through a variety of increasingly complex problems, extending their understanding of adding fractions to calculations that **exceed 1 as a mixed number**.

Partition and recombine

The children apply and extend their understanding of place value and **number bonds** (including the 'story' of 1), to partition and recombine calculations.

£1.35 + £0.85

£1.20 £0.15

£1.20 + (£0.15 + £0.85)

£1.20 + £1.00

= £2.20

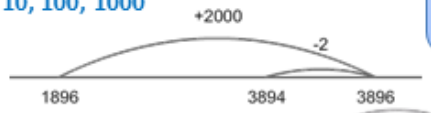
Bar modelling

Bar models are used to illustrate word problems, including fractions e.g.

Kelly purchased 5/9 meter of wire at the store. She used 1/3 metre in her art project. What is the length of the remaining piece?

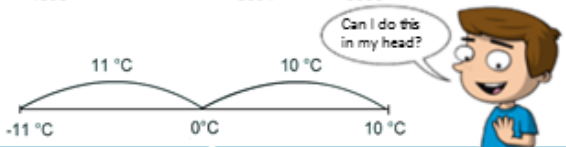
Near multiples of 10, 100, 1000

1896 + 1998
= 1896 + 2000 – 2
= 3894



Bridging zero

e.g. It is 21 degrees hotter is Sydney than Moscow. How hot is it in Sydney?



Counting on in tenths and hundredths

e.g. 1.7 + 0.55
= 1.7 + 0.3 + 0.2 + 0.05
= 2.25

Reorder

e.g.
4.7 + 5.6 – 0.7
= 4.7 – 0.7 + 5.6
= 4 + 5.6

Near doubles

e.g.
2.5 + 2.6
= double 2.5 +0.1
Or
= double 2.6 - 0.1

Count on

At this stage children only partition the second number when counting on

e.g. 5340 + 1280
= 5340 + 1000 + 200 + 80
= 6540 + 60 + 20
= 6620

Round and adjust

Round and adjust to create easier numbers to work with:

547 + 297

Round 297 to 300 by adding 3

547 + 300 = 847

then subtract 3 to get:

847 – 3 = 844

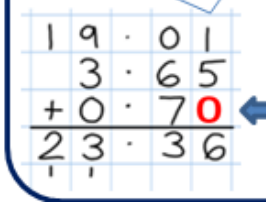
Compact Written Method

At this stage, the compact method is used to calculate with numbers **beyond 4-digits** and adding **several numbers with different numbers of digits**. In the context of money and measures, children practise adding decimals with **up to three decimal places**, including a **mix of whole numbers and decimals**, decimals with ***different* numbers of decimal places,** and **complements of 1** (for example, 0.83 + 0.17 = 1).

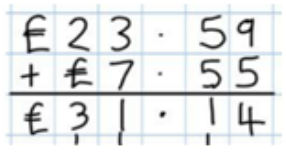
Say, '1 hundredth add 5 hundredths'

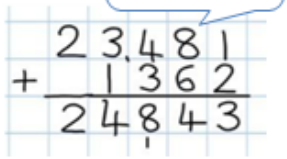
Decimal points are aligned in the same way as the other place value columns and must be in the same column in the answer.

Say, '8 tens add 6 tens' to reinforce place value.



Zero
Empty decimal places can be filled with a 0 to show the place value in each column.





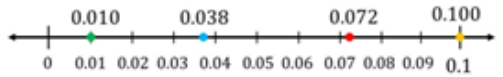
Digits are carried underneath.

Key skills:

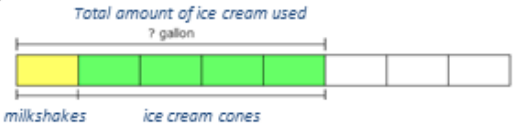
- Read, write, order and compare numbers to at least **1 000 000** (with **up to 3 decimal places**) and determine the value of each digit
- Round any number up to **1 000 000** to the nearest 10, 100, 1000, 10,000 and 100,000
- Count forwards or backwards with **positive and negative whole** numbers, including **through zero** and in steps of **powers of 10** for any given number up to **1 000 000**.
- Extend counting from year 4, using **decimals** and **fractions** including **bridging zero**, for example on a **number line**.
- Add whole numbers with **more than 4 digits**, including using the formal written method of column addition.
- Add numbers **mentally** with increasingly large numbers, and numbers with **up to 3 decimal places**, using and practising a range of **mental strategies**
- Use rounding and **estimation** to check answers and levels of accuracy
- Solve **multi-step problems** in contexts, deciding which operations and methods to use and why.

Fractions Children learn to:

- recognise and use **thousandths** and relate them to tenths, hundredths and **decimal equivalents.**
- **add fractions** extending to calculations that **exceed 1 as a mixed number**.
- **mentally** add and subtract **tenths**, and one-digit whole numbers and tenths.
- add fractions with the **same denominator** and **denominators that are multiples of the same number**



Allen used 1/8 gallon of ice cream to make milkshakes and 1/2 gallon of ice cream to make ice cream cones. How much ice cream did Allen use altogether?



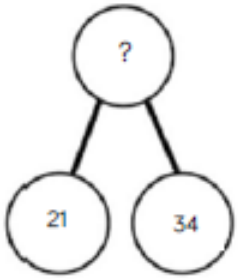
Mathematical Vocabulary

Pupils should read, spell and pronounce mathematical vocabulary correctly.

add, more, plus, and, make, altogether, total, equal to, equals, double, most, count on, number line, sum, tens, units, partition, addition, column, tens boundary, hundreds boundary, increase, vertical, expanded, compact, thousands, hundreds, digits, inverse, decimal places, decimal point, tenths, hundredths, thousandths

YEAR FIVE - Addition

Conceptual variation; different ways to ask children to solve 21 + 34



?	
21	34

Word problems:
In year 3, there are 21 children and in year 4, there are 34 children.
How many children in total?

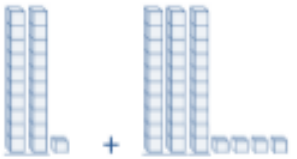
21 + 34 = 55. Prove it

21
+34
——

21 + 34 =

☐ = 21 + 34

Calculate the sum of twenty-one and thirty-four.



Missing digit problems:

10s	1s
10 10	1
10 10 10	?
?	5

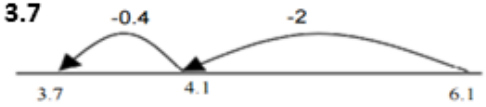
Add numbers with more than 4-digits

Children practise using the formal written methods of **columnar subtraction** with increasingly large numbers to aid fluency (e.g. 12 462 – 2 300 = 10 162). Place value counters are used to consolidate and secure understanding. They practise **subtracting fractions** to become fluent through a variety of increasingly complex problems, extending their understanding of subtracting fractions to calculations that **exceed 1 as a mixed number**.

Partitioning

The number line and informal jottings continue to be used to support mathematical thinking

6.1 – 2.4 = 3.7



£1.35 - £0.85

£1.35 - £0.35 – £0.50

£1.00 - £0.50 = £0.50

Bar modelling

Bar models are used to illustrate word problems, including fractions e.g.

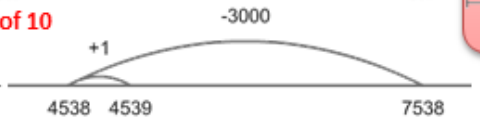
Kelly purchased 5/9 meter of wire at the store. She used 1/3 metre in her art project. What is the length of the remaining piece?

Near multiples of 10

7538 - 2999

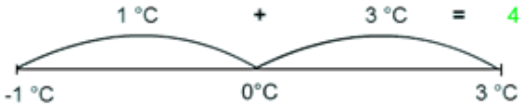
= 7538 – 3000 + 1

= 4539



Bridging zero

e.g. What is the difference in temperature between Glasgow and Winnipeg?



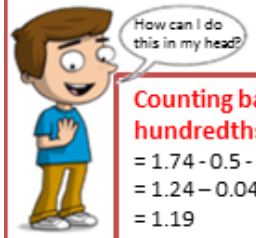
Reordering

e.g.

4.7 + 5.6 – 0.7

= 4.7 – 0.7 + 5.6

= 4 + 5.6



Counting back in tenths and hundredths

e.g. **1.74 - 0.55**

= 1.74 - 0.5 - 0.05

= 1.24 – 0.04 – 0.01

= 1.19

Counting back

At this stage children only partition the second number.

e.g. **5340 - 1280**

= 5340 - 1000 - 200 - 80

= 4140 – 40 - 40

= 4100 – 40 = 4060

Round and adjust

Round and adjust to create easier numbers to work with:

547 – 297

Round 297 to 300 by adding 3

547 - 300 = 247

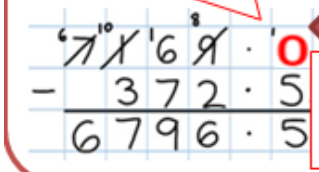
then add 3 to get:

247 + 3 = 250

Compact Written Method

At this stage, the compact method is used to calculate with numbers **beyond 4-digits**. In the context of money and measures, children practise subtracting decimals with **up to three decimal places**, including **whole numbers and decimals**, decimals with ***different*** **numbers of decimal places,** and **complements of 1** (for example, 1 - 0.83 = 0.17).

Say, '10 hundredths subtract 5 hundredths'

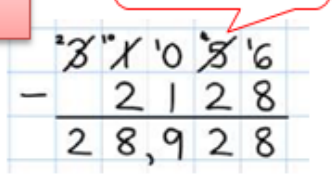


Zero

Empty decimal places can be filled with a 0 to show the place value in each column.

Decimal points are aligned in the same way as the other place value columns and must be in the same column in the answer.

Say, '4 tens, subtract 2 tens' to reinforce place value.



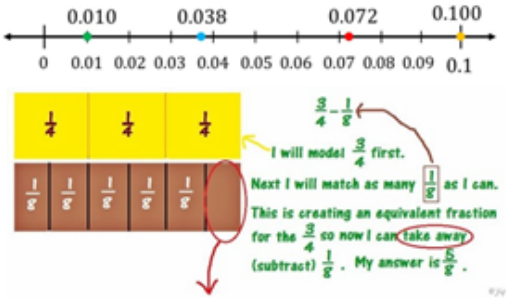
Subtraction Y5

Key skills:

- Read, write, order and compare numbers to at least **1 000 000** (with **up to 3 decimal places**) and determine the value of each digit
- Round any number up to **1 000 000** to the nearest 10, 100, 1000, 10,000 and 100,000
- Count forwards or backwards with **positive and negative whole** numbers, including **through zero** and in steps of **powers of 10** for any given number up to **1 000 000.**
- Extend counting from year 4, using **decimals** and **fractions** including **bridging zero,** for example, on a **number line.**
- Subtract whole numbers with **more than 4 digits**, including using the formal written method of column subtraction.
- Subtract numbers **mentally** with increasingly large numbers, and numbers with **up to 3 decimal places**, using and practising a range of **mental strategies**
- Use rounding and **estimation** to check answers and levels of accuracy
- Solve **multi-step problems** in contexts, deciding which operations and methods to use and why.

Fractions Children learn to:

- recognise and use **thousandths** and relate them to tenths, hundredths and **decimal equivalents.**
- **subtract fractions**
- **mentally** subtract **tenths**, and one-digit whole numbers and tenths.
- Subtract fractions with the **same denominator** and **denominators that are multiples of the same number**



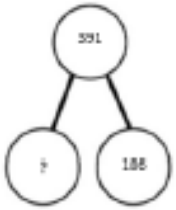
Mathematical Vocabulary

Pupils should read, spell and pronounce mathematical vocabulary correctly.

Take, take away, minus, subtract, subtraction, difference between, less than, more, total, equal to, equals, more, count on, decrease, least, how many left?, how much less is...?, column subtraction, inverse, hundreds, tens, ones, most, least, number line, partition, tens boundary, hundreds boundary, vertical, column, exchange, expanded, compact.

YEAR FIVE - Subtraction

Conceptual variation; different ways to ask children to solve 391 - 186



391	
186	?

Raj spent £391, Timmy spent £186.
How much more did Raj spend?

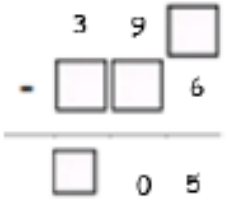
Calculate the difference between 391 and 186.

□ = 391 – 186

391
-186
———

What is 186 less than 391?

Missing digit calculations



Multiplication Y5

Multiply up to 4 digit numbers by 1- or 2-digit numbers

Children continue to use **short multiplication**, extending this to multiplying **4 digit numbers by a single digit**. They are introduced to **long multiplication** for multiplying by **2 digits**. They will apply all the multiplication tables and related division facts frequently, commit them to memory and use them confidently to make larger calculations.

Mental Methods

Mental methods introduced in previous year groups continue to be embedded and developed.

Distributive law

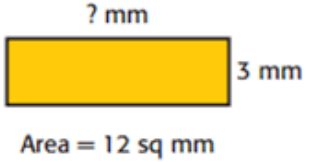
Children use the distributive law to partition 2-digit numbers, including decimals, to multiply by a 1-digit number mentally, expressing a(b + c) as ab + ac.
e.g. *6 × 27 as 6 × 20 (120) plus 6 × 7 (42)*
e.g. *6·3 × 7 as 6 × 7 (42) plus 0·3 × 7 (2·1)*

Measurement

Children will multiply and divide by powers of 10 in scale drawings and when converting between units of metric measure. (see division).

Area and algebra

Children will use the properties of rectangles to deduce related facts and find **missing lengths** or **areas.** Missing measures questions begin to be expressed algebraically, e.g. **4 + 2b = 20** for a rectangle of sides 2 cm and b cm and perimeter of 20cm.



Long multiplication

e.g. 123 x 45

Approximate

Children are taught to approximate first in order to minimise errors in calculations., for example, 123 x 45 is approximately 120 x 50 = 6000

Step 1

	1	2	3
x		4	5
	6	1	5
	1	1	

(123 x 5)

Digits are carried underneath.

Step 2

	1	2	3
x		4	5
	6	1	5
	1	1	
			0

Zero is used as a place holder because we are multiplying by tens.

Step 3

	1	2	3
x		4	5
	6	1	5
	1	1	
4	9	2	0
	1		

(123 x 40)

Step 4

	1	2	3
x		4	5
	6	1	5
	1	1	
4	9	2	0
	1		
5	5	3	5
1			

(615 + 4920)

Mathematical Vocabulary

Pupils should read, spell and pronounce mathematical vocabulary correctly.

group, groups of, times, array, altogether, column, row, sets, multiply, count, multiplied by, grid method, multiple, product, inverse, repeated addition, column, row, sets of, equal groups, times, once, twice, three times..., partition, grid method, multiple, value, square, factor, integer, decimal, short/long multiplication, decimal, square number, cube number, prime number, common factor, composite (non-prime) number, power of, **BIDMAS**

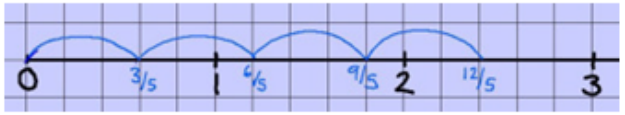
Key skills:

- Identify multiples and factors, including all factor pairs of a number, and common factors of two numbers.
- Know and use vocabulary of prime numbers, prime factors and composite (non-prime) numbers.
- Establish whether a number up to 100 is prime and recall prime numbers up to 19
- Multiply numbers up to 4 digits by a 1-or 2-digit number, using the formal written method, including long multiplication for two-digit numbers when children are confident in the other methods.
- Multiply and divide numbers mentally, drawing upon known facts.
- Multiply and divide whole numbers and those involving decimals by 10, 100 and 1,000
- Recognise and use square and cube numbers, and the notation for each (x^2, x^3) and use them to construct equivalence, for example, 4 x 35 = 2 x 2 x 35; 3 x 270 = 3 x 3 x 9 x 10 = 92 x 10).
- solve problems involving division including using their knowledge of factors and multiples, squares and cubes
- solve problems involving using all four operations, including understanding the meaning of the equals sign
- solve problems involving division, including scaling by simple fractions and problems involving simple rates.

Fractions

Children will begin to multiply proper fractions and mixed numbers by whole numbers ≤ 10, supported by materials and diagrams.

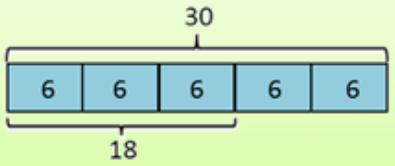
e.g. $4 \times {}^{3}/_{5} = {}^{12}/_{5} = 2\,{}^{2}/_{5}$



Bar modelling

Pupils use the bar model to illustrate problems and to assist children in determining the calculation required. The calculation steps should be written alongside the model.

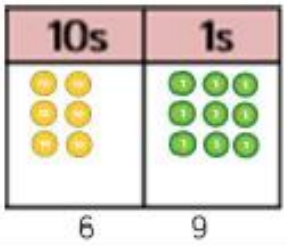
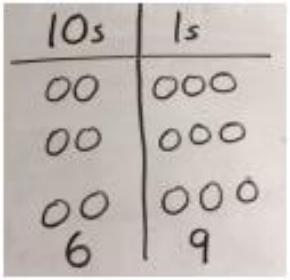
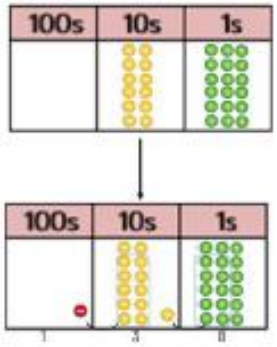
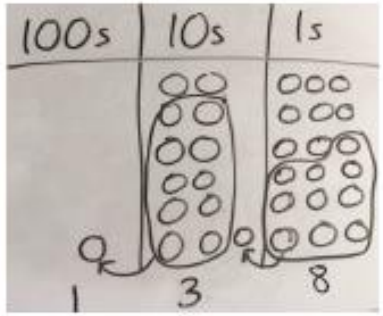
$\frac{3}{5}$ of **30** or $\frac{3}{5}$ x **30**



30 ÷ 5 = 6

6 x 3 = 18

YEAR FIVE - Multiplication

Concrete	Pictorial	Abstract
Formal column method with place value counters (base 10 can also be used.) 3 × 23  	Children to represent the counters pictorially.  	Children to record what it is they are doing to show understanding. 3 × 23 20 3 3 × 20 = 60 3 × 3 = 9 60 + 9 = 69 23 × 3 69
Formal column method with place value counters. 6 x 23  	Children to represent the counters/base 10, pictorially e.g. the image below.  	Formal written method 6 x 23 = 23 × 6 138 1 1
When children start to multiply 3d × 3d and 4d × 2d etc., they should be confident with the abstract: To get 744 children have solved 6 × 124. To get 2480 they have solved 20 × 124.		1 2 4 × 2 6 7 4 4 2 4 8 0 3 2 2 4 1 1 Answer: 3224

Division Y5

Divide 4-digit numbers by a single digit, including calculations with remainders

Children practise and extend their use of the formal written methods of **short division.**, being introduced to calculations that give rise to answers with **remainders.** They apply all the **multiplication tables** and **division facts** frequently, commit them to memory and use them confidently to make larger calculations. Place value counters are used to consolidate and secure understanding.

Multiplying by powers of 10

Pupils use multiplication and division as inverses to support the introduction of ratio in year 6, for example, by multiplying and dividing by powers of 10 in scale drawings or by multiplying and dividing by powers of a 1000 in converting between units such as kilometres and metres.

Mental Strategies

Children calculate mentally using the 'build up' method (where they 'build up' multiples to find the total amount being divided.

Written Method

The short division algorithm, is introduced in Year 5 with numbers up to 4 digits being divided by a 1-digit number.

Interpreting remainders

Children experience division in real life, problem solving contexts, where they must consider the meaning of the remainder and how to express it.

976 ÷ 8 = ?

100 x 8 = 800
20 x 8 = 160
2 x 8 = 16

So, *122* x 8 = 976
and 976 ÷ 8 = 122

$$\begin{array}{r} 115\ r4 \\ 5\overline{)5\,7^{2}9} \end{array}$$

As a fraction...

$$\begin{array}{r} 115\frac{4}{5} \\ 5\overline{)5\,7^{2}9} \end{array}$$

As a decimal...

$$\begin{array}{r} 115.8 \\ 5\overline{)5\,7^{2}9^{4}0} \end{array}$$

Doubling and halving

5 is half of 10, so
34 ÷ 5 is (34 ÷ 10) × 2

Jottings

Jottings (multiples) are used to support both mental and written methods. Children should be taught to use them flexibly, writing as much or as little down as required.

1 → 8
2 → 16
4 → 32

Key skills:

- Identify multiples and factors, including all factor pairs of a number, and common factors of two numbers.
- Know and use vocabulary of prime numbers, prime factors and composite (non-prime) numbers.
- Establish whether a number up to 100 is prime and recall prime numbers up to 19
- Multiply numbers up to 4 digits by a 1-or 2-digit number, using the formal written method, including long multiplication for two-digit numbers when children are confident in the other methods.
- Multiply and divide numbers mentally, drawing upon known facts.
- Divide numbers up to 4 digits by a one-digit number using the formal written method of short division and interpret remainders appropriately for the context
- Multiply and divide whole numbers and those involving decimals by 10, 100 and 1,000
- Recognise and use square and cube numbers, and the notation for each (x^2, x^3) and use them to construct equivalence

for example, 4 x 35 = 2 x 2 x 35; 3 x 270 = 3 x 3 x 9 x 10 = 92 x 10).

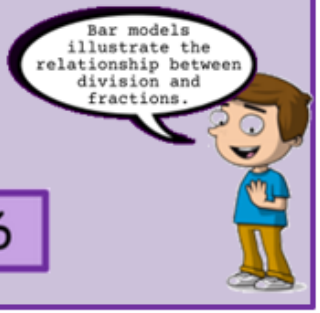
- solve problems involving division including using their knowledge of factors and multiples, squares and cubes
- solve problems involving using all four operations, including understanding the meaning of the equals sign
- solve problems involving division, including scaling by simple fractions and problems involving simple rates.

Mathematical Vocabulary

Pupils should read, spell and pronounce mathematical vocabulary correctly.

share, share between, share equally, group, groups of, lots of, divide, divided by, divided into, division, left, left over, grid method, multiple, product, inverse, short division, 'carry', remainder, factor, quotient, divisor, square number, cube number, prime number, common factor, composite (non-prime) number, power of

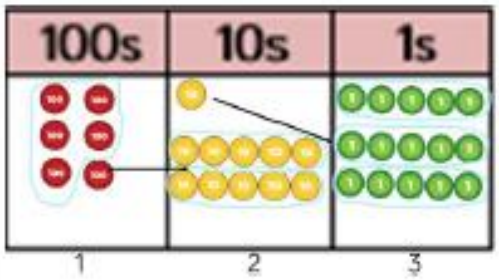
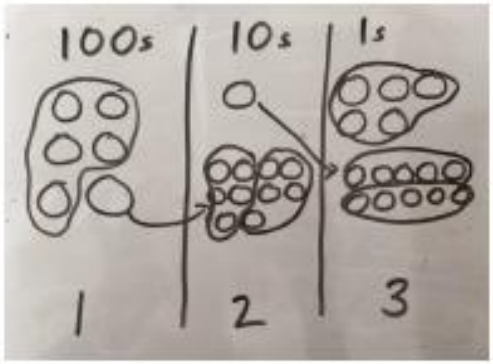
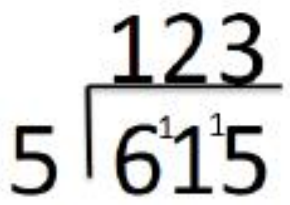
Finding $\frac{1}{5}$ of an amount is the same as dividing that amount by 5.



So $\frac{1}{5}$ of 30 = 6

30 ÷ 5 = 6

YEAR FIVE - Division

Short division using place value counters to group. 615 ÷ 5  1. Make 615 with place value counters. 2. How many groups of 5 hundreds can you make with 6 hundred counters? 3. Exchange 1 hundred for 10 tens. 4. How many groups of 5 tens can you make with 11 ten counters? 5. Exchange 1 ten for 10 ones. 6. How many groups of 5 ones can you make with 15 ones?	Represent the place value counters pictorially.  	Children to the calculation using the short division scaffold. 

Remember, children should be encouraged to give remainder answers as a mixed number answer and a decimal

E.g. 432 ÷ 5 = 86 r2

This could be given as: 82.4 or 86 2/5

Addition Y6

Add several numbers of increasing complexity

Children use the whole number system, practising addition with larger numbers, using the **formal written methods** of columnar addition. They undertake **mental calculations** with increasingly large numbers and more complex calculations. Pupils use knowledge of the order of operations (BIDMAS) when one or more operation is included in the calculation.

Partitioning and Recombining

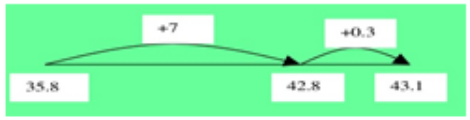
Children partition in different ways and in a variety of different contexts, developing and applying their knowledge of **number bonds** and **place value**, to include decimal fractions e.g.

35.8 + 7.3

= 35.8 + 7 + 0.3

= 42.8 + 0.3

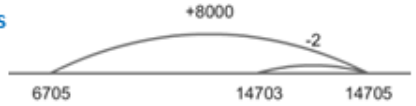
= 43.1



Adding near multiples

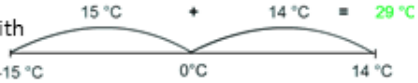
e.g.

6705 + 7998 = 14703



Negative numbers

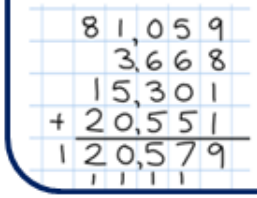
are used in context and with intervals across zero.

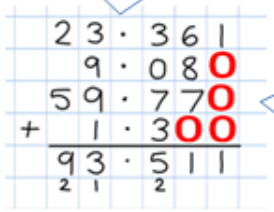


Compact Written Method

Children add **several numbers** with **more than 4 digits** and **different numbers of decimal places** (including money and measures):

Decimal points should line up in columns.





'Empty decimal places can be filled with a 0 to show the place value in each column.

BIDMAS (brackets, indices, division *and* multiplication, addition *and* subtraction. Order of operations models must remind learners that multiplication and division are of equal weighting and should be calculated left to right, as are addition and subtraction

e.g. **5 + 3 x 6 = 23** **(5 + 3) x 6 = 48**

Algebra

Children extend their investigations into missing number/ digit problems to include two unknowns, each representing a different value e.g.

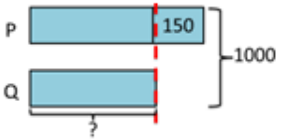
A = 3

A + A = Z + Z + A

What is the value of Z?

What if A = 28? What if A = 21?

p and q each stand for whole numbers. p + q = 1000 and p is 150 greater than q. Work out the values of p and q



3a + 5 = 17

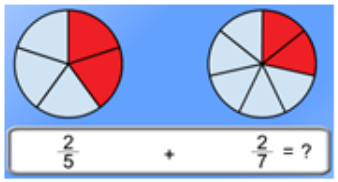
Key skills:

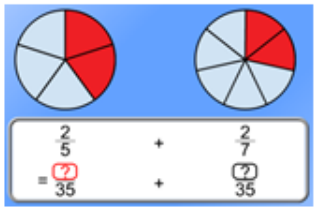
- Read, write, order and compare numbers up to 10 000 000 and determine the value of each digit
- Identify the value of each digit in numbers given to three decimal places
- Round any whole number to a required degree of accuracy
- Use negative numbers in context, and calculate intervals across zero
- Solve problems which require answers to be rounded to specified degrees of accuracy
- Solve number and practical multi-step problems in contexts, deciding which operations and methods to use and why.
- Use estimation to check answers to calculations and determine, in the context of a problem, an appropriate degree of accuracy for example, to the nearest 10, 20, 50 etc., but not to a specified number of significant figures. .

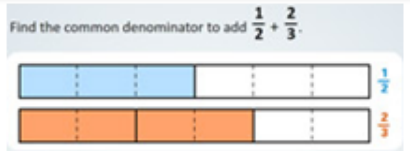
Fractions

- Pupils should practise, use and understand the **addition** of fractions with **different denominators** by identifying equivalent fractions with the same denominator.
- They should start with fractions where the denominator of one fraction is a multiple of the other and progress to varied and increasingly complex problems.
- Recall and use **equivalences** between **simple fractions, decimals and percentages**, including in different contexts.
- Use **common factors** to simplify fractions;
- Use **common multiples** to express fractions in the same denominator.

Children will add fractions with different denominators and mixed numbers, using the concept of equivalent fractions







Common multiples are used to express fractions in the same denominator.

Mathematical Vocabulary

Pupils should read, spell and pronounce mathematical vocabulary correctly.

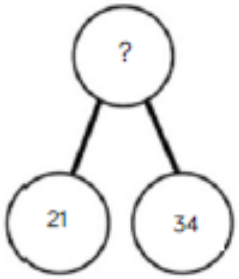
add, addition, more, plus, altogether, total, equal to, equals, more, and, make, sum, double, count on, increase, column, hundreds, thousands, tens, ones, most, least, number line, partition, recombine tens boundary, hundreds boundary, vertical, exchange, expanded, compact perimeter, digits, inverse, decimal places, decimals point, tenths, hundredths, thousandths, numerator, denominator.

Mental strategies

continue to be practised and refined. Place value counters can be used to consolidate understanding of place value and exchange.

YEAR SIX - Addition

Conceptual variation; different ways to ask children to solve 21 + 34



?	
21	34

Word problems:
In year 3, there are 21 children and in year 4, there are 34 children.
How many children in total?

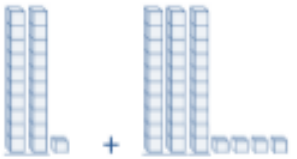
21 + 34 = 55. Prove it

21
+34
——

21 + 34 =

☐ = 21 + 34

Calculate the sum of twenty-one and thirty-four.



Missing digit problems:

10s	1s
	?
?	5

Subtraction Y6

Subtract numbers of increasing complexity

Children use the whole number system, practising subtraction with larger numbers and decimals, using the **formal written methods** of columnar subtraction. They undertake **mental calculations** with increasing large numbers and more complex calculations. Pupils use knowledge of the order of operations (BIDMAS) when one or more operation is included in the calculation.

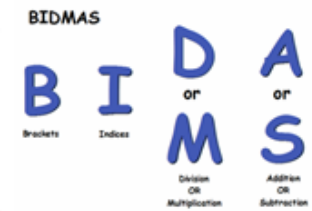
Partitioning and Recombining

Children partition in different ways and in a variety of different contexts, developing and applying their knowledge of **number bonds** and **place value**, to include decimal fractions e.g.

1.74 - 0.55
= 1.74 - 0.5 - 0.05
= 1.24 – 0.04 – 0.01
= 1.19

Mental strategies

continue to be practised and refined. Place value counters can be used to consolidate understanding of place value and exchange.

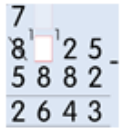


BIDMAS (brackets, indices, division multiplication, addition *and* subtraction. Order of operations models must remind learners that multiplication and division are of equal weighting and should be calculated left to right, as are addition and subtraction

e.g. **5 - 3 x 6 = -13** **(5 - 3) x 6 = 12**

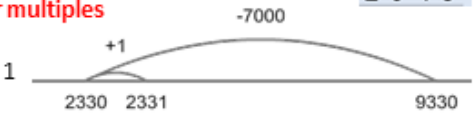
Solve missing number/ digit problems

Children investigate missing number/digit problems



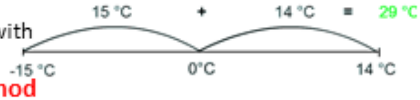
Subtracting near multiples

e.g. **9330 - 6999**
= 9330 – 7000 + 1
= 2331



Negative numbers

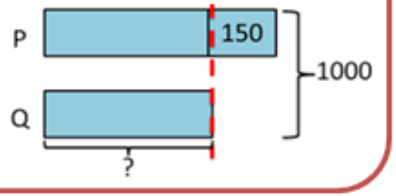
are used in context and with intervals across zero.



Algebra

Children extend their investigations into missing number/ digit problems to include two unknowns, each representing a different value e.g.

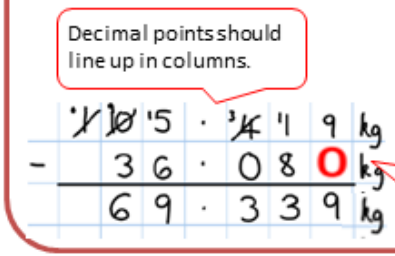
p and q each stand for whole numbers. p + q = 1000 and p is 150 greater than q. Work out the values of p and q

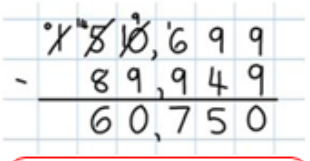


Compact Written Method

Children subtract **numbers** with **more than 4 digits** and **different numbers of decimal places** (including money and measures):

Decimal points should line up in columns.





Empty decimal places can be filled with a 0 to show the place value in each column.

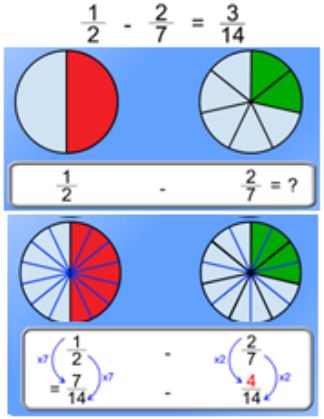
Key skills:

- Read, write, order and compare numbers up to **10 000 000** and determine the value of each digit
- Identify the value of each digit in numbers given to **three decimal places**
- Round any whole number to a required degree of accuracy
- Use **negative numbers** in context, and calculate **intervals across zero**
- Solve problems which require answers to be **rounded to specified degrees of accuracy**
- Solve number and practical **multi-step problems** in contexts, deciding which operations and methods to use and why.
- Use **estimation** to check answers to calculations and determine, in the context of a problem, an **appropriate degree of accuracy** for example, to the nearest 10, 20, 50 etc., but not to a specified number of significant figures.

Fractions

- Pupils should practise, use and understand the **subtraction** of fractions with **different denominators** by identifying equivalent fractions with the same denominator.
- They should start with fractions where the denominator of one fraction is a multiple of the other and progress to varied and increasingly complex problems.
- Recall and use **equivalences** between **simple fractions, decimals and percentages**, including in different contexts.
- Use **common factors** to simplify fractions;
- Children will subtract fractions with different denominators and mixed numbers, using the concept of equivalent fractions.

Common multiples are used to express fractions in the same denominator.



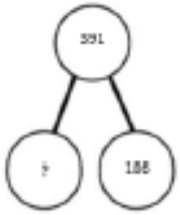
Mathematical Vocabulary

Pupils should read, spell and pronounce mathematical vocabulary correctly.

add, addition, more, plus, altogether, total, equal to, equals, more, and, make, sum, double, count on, increase, column, hundreds, thousands, tens, ones, most, least, number line, partition, recombine tens boundary, hundreds boundary, vertical, exchange, expanded, compact perimeter, digits, inverse, decimal places, decimals point, tenths, hundredths, thousandths, numerator, denominator.

YEAR SIX - Subtraction

Conceptual variation; different ways to ask children to solve 391 - 186



391	
186	?

Raj spent £391, Timmy spent £186.
How much more did Raj spend?

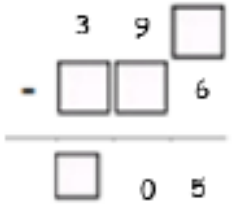
Calculate the difference between 391 and 186.

□ = 391 – 186

391
-186
———

What is 186 less than 391?

Missing digit calculations



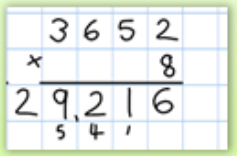
Multiplication Y6

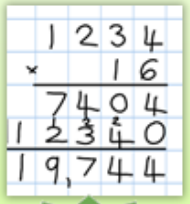
Multiply numbers of increasing complexity

Children use the whole number system, practising multiplication with larger numbers, using the **formal written methods** of long and short multiplication. They undertake **mental calculations** with increasingly large numbers and more complex calculations. Children continue to use all the multiplication tables to calculate mathematical statements in order to maintain their fluency. They use knowledge of the order of operations (BIDMAS) when one or more operation is included in the calculation; for example, 2 + 1 x 3 = 5 and (2 + 1) x 3 = 9.

Formal Written Method

Children will continue to use short and long division methods to multiply multi-digit numbers up to 4 digits by one- and two-digit whole numbers.





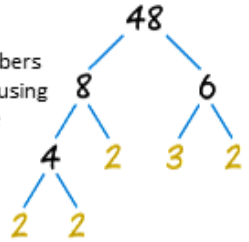
Approximate

Algebra

Children extend their investigations into missing number/ digit problems. *recognise when it is possible to use **formulae** for area and volume of shapes and* generate and describe linear number sequences. **4n+1**

Prime factors

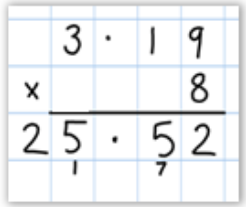
Children deepen their understanding of prime numbers and prime factors, including using factor trees to identify prime factors of numbers.



48 = 2 x 2 x 2 x 2 x 3

Multiplication with decimals

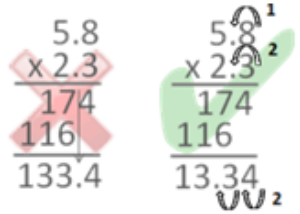
Children are introduced to multiplication of numbers with up to 2 decimal places by one- and two-digit whole numbers. They begin with simple cases, such as: 0.4 x 2 = 0.8 before moving on to more complex problems e.g.



Children use rounding to approximate before carrying out calculations, e.g. 72 x 3.8 is approximately 70 x 4 = 280. They then use the approximation to check the reasonableness of their answer.

Common errors

Children identify common errors and discuss how these may come about e.g. bringing the decimal point down (as done when adding/ subtracting) instead of counting the decimal places and using the total in the answer.

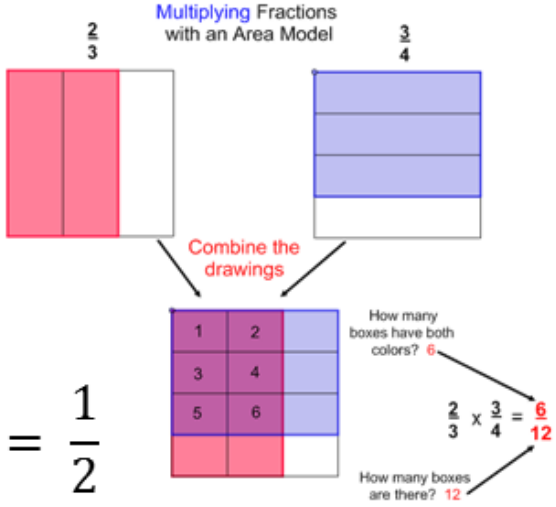


Key skills:

- Read, write, order and compare numbers up to 10 000 000 and determine the value of each digit in numbers given to three decimal places and multiply and divide numbers by 10, 100 and 1000 giving answers up to three decimal places
- Round any whole number to a required degree of accuracy, for example, to the nearest 10, 20, 50 etc., but not to a specified number of significant figures.
- Use negative numbers in context, and calculate intervals across zero
- Solve number and practical problems that involve all of the above
- Multiply multi-digit numbers up to 4 digits by a two-digit whole number using formal written method of long multiplication.
- Perform mental calculations, including with mixed operations and large numbers
- Identify common factors, common multiples and prime numbers
- Use their knowledge of the order of operations to carry out calculations involving the four operations
- Solve problems involving addition, subtraction, multiplication and division
- Use estimation to check answers to calculations and determine, in the context of a problem, an appropriate degree of accuracy

Fractions

- Children will use images to develop an understanding of multiplication of fractions
- They will multiply simple pairs of proper fractions, writing the answer in its simplest form for example,



$$\frac{2}{3} \times \frac{3}{4} = \frac{6}{12} = \frac{1}{2}$$

Mathematical Vocabulary

Pupils should read, spell and pronounce mathematical vocabulary correctly.

group, groups of, times, array, altogether, column, row, sets, multiply, count, multiplied by, grid method, multiple, product, inverse, repeated addition, column, row, sets of, equal groups, times, once, twice, three times..., partition, grid method, multiple, value, square, factor, integer, decimal, short/long multiplication, decimal, square number, cube number, prime number, common factor, composite (non-prime) number, power of, BIDMAS

YEAR SIX - Multiplication

Conceptual variation; different ways to ask children to solve 6 × 23

23	23	23	23	23	23

?

Mai had to swim 23 lengths, 6 times a week.
How many lengths did she swim in one week?

With the counters, prove that 6 x 23 = 138

Find the product of 6 and 23

6 × 23 =

☐ = 6 × 23

6	23
× 23	× 6
___	___

What is the calculation?
What is the product?

100s	10s	1s
		

Division Y6

Divide numbers of increasing complexity

Children use the whole number system, practising division with larger numbers, using the **formal written methods** of short and long division. They undertake **mental calculations** with increasingly large numbers and more complex calculations. Pupilsuse knowledge of the order of operations (BIDMAS) when one or more operation is included in the calculation.

Written Methods

The short division algorithm, introduced in Year 5, continues to be practised with numbers with at least 4 digits (see below). Children use written division to solve problems where the answers, expressed as remainders, fractions or decimals, are given to a required degree of accuracy and interpreted in context.

Answers as remainders...

$$8\overline{)5^{5}3^{3}0^{2}9} = 0663\text{ r }5$$

1 → 8
2 → 16
4 → 32

As decimals...

$$8\overline{)6^{6}4\ 9^{1}7\cdot0^{2}0^{4}0} = 8125$$

Long division

Children progress to dividing multi-digit numbers (up to 4 digits) by a two digit whole number using the formal written method of long division.

$$28\overline{)1653} = 59\frac{1}{28}$$

$$-\ 140$$

$$253$$

$$-\ 252$$

$$1$$

5 x 28 = 140
9 x 28 = 252

59

Mental Strategies

Children use the 'build up' method (where the y 'build up' multiples to find the total amount being divided, to calculate mentally.

How many 13s are there in 1,937?

1937 ÷ 13 = ?

100 x 13 = 1300

40 x 13 = 520

9 x 13 = 117

So, 149 x 13 = 1937

and 1937 ÷ 13 = 149

Jottings

Children use jottings (multiples) to support both mental and efficient methods. They should be taught to use them flexibly, writing as much or as little down as required.

Mathematical Vocabulary

Pupils should read, spell and pronounce mathematical vocabulary correctly.

share, share between, share equally, group, groups of, lots of, divide, divided by, divided into, division, left, left over, grid method, multiple, product, inverse, short division, 'carry', remainder, factor, quotient, divisor, square number, cube number, prime number, common factor, composite (non-prime) number, power of, BIDMAS

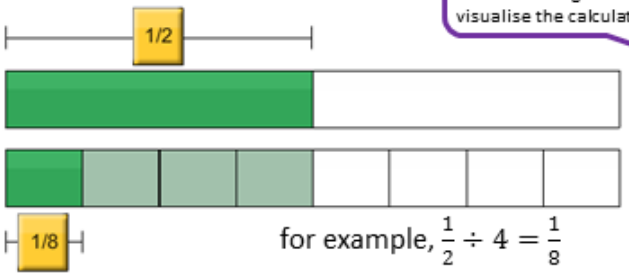
Key skills:

- Read, write, order and compare numbers up to 10 000 000 and determine the value of each digit in numbers given to three decimal places, and multiply and divide numbers by 10, 100 and 1000 giving answers up to three decimal places.
- Round any whole number to a required degree of accuracy, for example, to the nearest 10, 20, 50 etc., but not to a specified number of significant figures.
- Use negative numbers in context, and calculate intervals across zero
- Solve number and practical problems that involve all of the above
- Divide numbers up to 4 digits by a two-digit whole number using the formal written method of long division, and interpret remainders as whole number remainders, fractions, or by rounding, as appropriate for the context
- Divide numbers up to 4 digits by a two-digit number using the formal written method of short division where appropriate, interpreting remainders according to the context
- Perform mental calculations, including mixed operations and large numbers
- Identify common factors, common multiples and prime numbers
- Use their knowledge of the order of operations to carry out calculations involving the four operations
- Solve problems involving addition, subtraction, multiplication and division
- Use estimation to check answers to calculations and determine, in the context of a problem, an appropriate degree of accuracy

Fractions

- Children learn to associate a fraction with division including when calculating remainders, and calculate decimal fraction equivalents for simple fractions e.g. 0.375 = $\frac{3}{8}$
- *Common multiples are used to express fractions in the same denomination.*
- *Common factors are used to simplify fractions;*

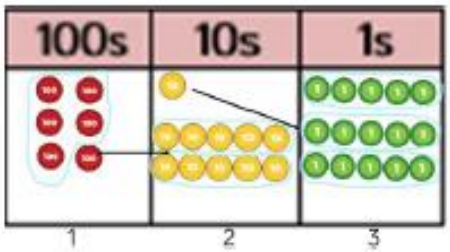
Children learn to divide proper fractions by whole numbers using a bar model to visualise the calculation.



for example, $\frac{1}{2} \div 4 = \frac{1}{8}$

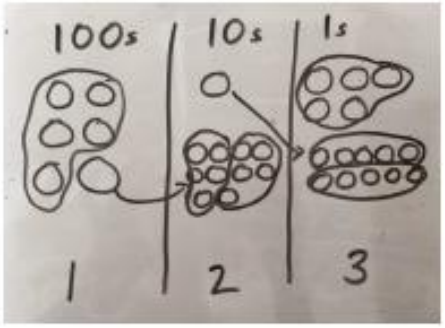
YEAR SIX - Division

Short division using place value counters to group.
615 ÷ 5



1. Make 615 with place value counters.
2. How many groups of 5 hundreds can you make with 6 hundred counters?
3. Exchange 1 hundred for 10 tens.
4. How many groups of 5 tens can you make with 11 ten counters?
5. Exchange 1 ten for 10 ones.
6. How many groups of 5 ones can you make with 15 ones?

Represent the place value counters pictorially.

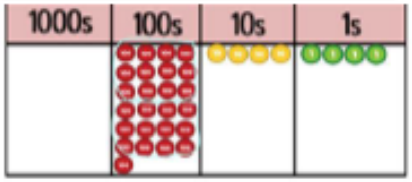


Children to the calculation using the short division scaffold.

$$5\overline{)615} = 123$$

Long division using place value counters
2544 ÷ 12

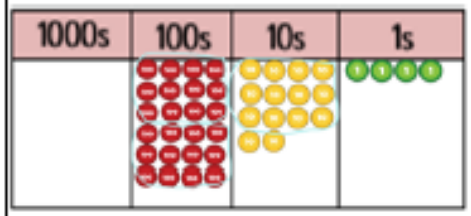
We can't group 2 thousands into groups of 12 so will exchange them.



We can group 24 hundreds into groups of 12 which leaves with 1 hundred.

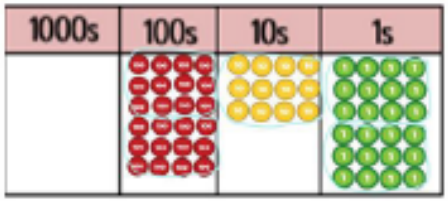
$$\begin{array}{r} 02 \\ 12\overline{)2544} \\ \underline{24} \\ 1 \end{array}$$

YEAR SIX - Division



After exchanging the hundred, we have 14 tens. We can group 12 tens into a group of 12, which leaves 2 tens.

$$\begin{array}{r} 0\,2\,1 \\ 12\overline{)\,2544} \\ 24 \\ \hline 14 \\ 12 \\ \hline 2 \end{array}$$



After exchanging the 2 tens, we have 24 ones. We can group 24 ones into 2 group of 12, which leaves no remainder.

$$\begin{array}{r} 0\,2\,1\,2 \\ 12\overline{)\,2544} \\ 24 \\ \hline 14 \\ 12 \\ \hline 24 \\ 24 \\ \hline 0 \end{array}$$

Conceptual variation; different ways to ask children to solve 615 ÷ 5

Using the part whole model below, how can you divide 615 by 5 without using short division?

I have £615 and share it equally between 5 bank accounts. How much will be in each account?

615 pupils need to be put into 5 groups. How many will be in each group?

$$5\overline{)\,615}$$

615 ÷ 5 =

□ = 615 ÷ 5

What is the calculation?
What is the answer?